



Structural Wind Loading Study

Centre Block, Parliament Hill

Ottawa, Ontario

REPORT: GWE 15-032-HFPI

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EXECUTIVE SUMMARY

This report describes a structural wind loading study for the Centre Block on Parliament Hill, Ottawa, Ontario, using the high-frequency pressure integration method. The study involves wind tunnel measurements of structural wind loads in terms of integrated wind pressure over the surface of a 1:250 scale model of the Centre Block and Peace Tower, combined with dynamic properties of the Peace Tower and also interpretation of the local wind climate. The results of the procedure include predicted base shear forces, overturning moments, and torsional moments for the Centre Block and Peace Tower, the effective static loads in the forms of floor-by-floor loads and pressure diagrams for the Peace Tower and Centre Block respectively, as well as building motion data of the Peace Tower in the form of accelerations. The information is useful for the structural design of the building and evaluation of occupant comfort in the tower with respect to its motion.

Predicted 50-year peak base moments and shears of the Peace Tower and mean (static) base forces of the Centre Block are summarized in the following tables, and repeated in the main body of the report, along with the detailed effective static wind loads. Forces and moments are referenced to the coordinate system illustrated in Figures 2a and 2b, following a context plan in Figure 1. Figure 3 presents the statistical model of the Ottawa wind climate. Code derived wind loads for the Peace Tower and Centre Block are provided for comparison in the tables below, which are seen to be larger than the more accurate wind tunnel results.

PEACE TOWER

	Peak Building Base Moments ($\times 10^5$ kN-m)			Peak Base Shears ($\times 10^3$ kN)	
	Mx	My	Mz	Fx	Fy
Wind Tunnel Test	0.43	0.44	0.0098	0.96	0.91
Code Estimation	0.59	0.60	-	1.17	1.16

CENTRE BLOCK

	Mean (Static) Base Torque ($\times 10^5$ kN-m)	Mean (Static) Base Shears ($\times 10^3$ kN)	
	Mz	Fx	Fy
Wind Tunnel Test	0.42	1.3	1.9
Code Estimation	-	1.3	2.4

Lateral acceleration and torsional velocity for the 10-year return period at the top occupied level of the Peace Tower and for a structural damping ratio of 0.020 are summarized in the following table. The accelerations are acceptable, falling below the industry guideline of 20 milli-g. The 10-year return period torsional velocity is also acceptable, based on the industry guidelines of 3.0 milli-radians/sec.

10-year Return Period Peak Resultant Acceleration (milli-g)	10-year Return Period Peak Torsional Velocity (milli-rad/sec)
12.3	0.59

All forces and moments in this report represent specified loads, which shall be increased by the usual safety factors, as required by the Ontario Building Code (OBC 2012) and structural design standards.

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Halsall Associates

1. INTRODUCTION

Gradient Wind Engineering Inc. (GWE) was retained by Halsall Associates Consulting Engineers on behalf of Public Works and Government Services Canada to undertake a detailed structural wind load study for the Peace Tower and Centre Block on Parliament Hill, Ottawa, Ontario. This report summarizes the methodology, results and recommendations related to the structural wind loading investigation for the study building using the High Frequency Pressure Integration (HFPI) technique. Our work is based on architectural drawings and structural information of the study building provided by Halsall in April and May respectively of 2015. Surrounding topography, street layouts and building massing information were obtained from the City of Ottawa, as well as recent site imagery.

2. TERMS OF REFERENCE

The focus of this structural study is the Centre Block and Peace Tower on Parliament Hill, Ottawa. The site is situated at the north centre of the Canadian Parliamentary Complex on Parliament Hill, bounded by Wellington Street to the south and the Ottawa River to the north.

The Centre Block features a Gothic style 6-storey building with a symmetric building plan with dimensions of approximately 144 meters (m) by 75 m in the east-west and north-south directions respectively. The building has sloped roofs around its perimeter and two courtyards at the east and west sides of the building, which are symmetrically located along the centre line of the building. The building also features two towers with square plan dimension of 5.2 m by 5.2 m at the northwest and northeast corners. Immediately to the south and connected to Centre Block, is the Peace Tower, which rises approximately 92 m above grade to the top of the structure, on a 12.2 m square floor plate dimension. Located to the north of the Centre Block and overlooking the Ottawa River at approximately 45 m on a steep promontory is the Library of Parliament, which features a circular building with a cone-shaped roof articulated with multiple spires on two levels, and gable dormers at mid-height between the spires. Based on the request for proposal, the current study does not include the wind loads on the separate Library structure.

Figures 1 and 2 illustrate an extended context plan and the reference coordinate system, respectively, while Photographs 1 through 4 illustrate the wind tunnel model, complete with the study building and surroundings.

3. OBJECTIVES

This study was commissioned as part of an overall structural upgrading for earthquake resistance and wind resistance. Hence, the principal objectives of this study are to determine the overall peak wind loads for structural design of the Peace Tower and Centre Block, and to assess the overall building accelerations that affect visitor comfort at the top of the Peace Tower.

4. STUDY METHODOLOGY

The approach used to quantify structural wind loads on the tower is the high-frequency pressure integration method. Using a 1:250 scale model of the study site and its surroundings within a full-scale diameter of 525 m, data collected from wind tunnel testing is combined with the dynamic properties of the building and a statistical model of the local wind climate. The analysis generates predictions of base shear forces, overturning and torsional moments, as well as the variation of wind forces as a function of building height.

4.1. Consideration of the Local Wind Climate

Hourly meteorological data from Ottawa International Airport, covering a period of over 40 years, were analyzed to obtain a statistical model of wind speed and direction for subsequent analysis. Figure 3 illustrates three contours representing three probability levels superimposed on a polar grid of wind speeds in meters per second (m/s), at an anemometer height of 10 m above grade. The three contours represent the wind speed occurring once-per-year (innermost contour), once-in-ten-year, and once-in-fifty-year (outermost contour). The 1-in-50-year contour is recommended for structural design. The preferred wind directions can be identified as the angular position where the given contour has the largest radial distance from the centre. For Ottawa, the most common winds occur from the west-northwesterly directions, followed by those from the southwesterly and east-northeasterly directions. This information was interpreted for the study site, based on consideration of the topography and surface roughness characteristics of the airport and the project site as a function of wind direction.

The statistical model of the Ottawa wind climate was calibrated to give a 50-year return period dynamic pressure of 0.41 kPa for strength design, considering the requirements of the Ontario Building Code (OBC 2012). Building accelerations were determined for 1-year and 10-year return periods by directly using the measured wind speeds without any adjustments to consider real on-site wind conditions. Appendix A describes how natural wind flowing over the earth's surface is simulated in the wind tunnel.

4.2. High-Frequency Pressure Integration Method

The high-frequency pressure integration (HFPI) method is a technique by which wind loads are measured as integrated pressure over the building envelope of a tall building or other flexible structure. The technique is used to formulate mean and fluctuating (quasi-static) values of the modal loads, base overturning moments, shear forces and torque, based solely on the shape of the building. Using these basic quantities combined with building mass and modes of vibration, it is then possible to infer the wind load distribution with height, as well as the building deflections and accelerations at selected heights. This information is used for the design of the structural system and to assess occupant comfort with respect to building motion.

In the HFPI method, the instantaneous forces are measured as integrated pressure over the exterior surface of a stiff (high-frequency) model of the study building that incorporates the influence of its geometric shape and the combined turbulent signature of the surrounding buildings. In effect, the model and the wind tunnel represent the complexities of wind flow over the site that cannot be achieved by any form of computational or analytical simulation. Following wind tunnel testing, the measured loads are combined analytically with the dynamic properties of the full-scale building to determine the wind-induced responses. In this way, any set of structural properties can be evaluated with the same basic wind tunnel data, provided the shape of the building does not change. Appendix B provides the mathematical background to the HFPI method.

4.3. Wind Tunnel Measurements

Wind tunnel measurements for this project were conducted in GWE's wind tunnel facility. A geometric model of the building among its surroundings was installed in the wind tunnel. The study model was fabricated from a dense polymer and instrumented with 362 pressure taps to envelop the exterior surface of the Peace Tower and the Centre Block with 8 pressure integration rings of 137 pressure taps and 225 pressure taps, respectively, as shown in Photograph 3. Each pressure tap consists of a small diameter (1.07 millimeters or 0.042 inches) stainless steel tube connected by flexible tubing to a miniature multi-port pressure scanner, capable of sampling input pressures at a rate of 500 samples per second. Testing is conducted for each 10° interval for the full compass azimuth using an automated turntable at a sampling rate of 500 samples per second for a period of 90 seconds, which correspond to approximately two-hours in full-scale.

The information is analyzed off-line to obtain spectra and peak, mean and root-mean-square (rms) values of wind loads including modal loads, base shear forces, moments and torque for each wind angle based on the HFPI method as described in detail in Appendix B. A spectrum is a mathematical tool used to interpret the energy contained in a signal such as wind speed or force as a function of frequency.

Wind exposures in full-scale are created in model-scale for each wind direction by representing existing and planned buildings within a radius of 263 m of the study site and generic turbulence elements beyond this distance along the length of the wind tunnel. Turbulence generators include roughness blocks along the floor and spires at the wind tunnel entrance. The study model and the surrounding buildings are illustrated in Photographs 1 to 3.

The wind tunnel study undertaken for this project meets or exceeds the requirements of 'Wind Tunnel Studies of Buildings and Structures', ASCE Manual 7 Engineering Practice Note 67.

4.4. Evaluation of Structural Responses

The modal loads represented by spectra for each wind angle are combined with the structural properties of the building to determine the actual responses. The procedure requires multiplication of the spectrum value at each frequency with the mechanical admittance function, incorporating the natural frequency and damping for a given mode of vibration, as described in Appendix B. The mass distribution of each floor is used to interpret the distribution of base loads as a function of height (see Appendix C). Table 1 tabulates the mass properties for the building. Analyses are performed for two levels of structural damping of 2.0% and 1.5% to provide wind load data for strength design and serviceability design, respectively. Colour separations have been used to improve the readability of the Table, and are not related to the interpretation of the conditions.

4.5. Building Accelerations – Peace Tower

Building responses, determined in accordance with the noted procedure, can also be used to obtain information on building motion. For tall buildings, the quantity of interest to occupants is the acceleration of the top occupied level. The total acceleration comprises a weighted combination of accelerations in each of the principal sway directions and torsion, and is represented by the following formula:

$$\hat{a}^2 = g_p (\alpha a_x'^2 + \beta a_y'^2 + \gamma a_z'^2)$$

Where a'_x, a'_y, a'_z are the root-mean-square (rms) of acceleration in each of the principal directions and torsion, \hat{a} is the peak acceleration, and g_p is the peak factor which converts the rms values to peaks. α, β and γ are correlation coefficients necessary to account for the lack of perfect correlation among the three components. Typically, among the three correlation coefficients, the one corresponding to the maximum response is equal to 1.0, while the remaining two coefficients are equal to a value between 0.5 and 0.7.

It should be noted that the criteria for acceptable levels of building motions in the form of sway and torsional accelerations are subject to variability due to a number of human physiological factors and state of mind. Hence, the criterion for total acceleration discussed in the results section is based on the assumption that a small percentage of occupants may find even limited levels of building motion to be objectionable.

4.6. Wind Load Distribution with Simultaneous Loads

4.6.1. Peace Tower

The principal effect of turbulent wind on a flexible structure, such as the Peace Tower, is to create motion, which translates into inertial forces due to the building's own mass. For structural design, the dynamic origin of wind loading can be represented by equivalent static wind loads having the same peak effect. Hence, the distribution of lateral forces with height in each of the two principal axes is obtained from the peak base moments, base shear forces, the mode shape and the mass of each floor. This procedure is described in Appendix C. Base torque is obtained simply by offsetting one of the sets of lateral forces from the centre of stiffness.

Experience and testing evidence demonstrates that the peak load effects in each axis direction will not occur at the same time. As a result, the peak effect on a structural member, such as a corner column or the corner section of a shear wall, will be some fraction of the loads obtained along each axis. Consideration of the loading effects, with respect to location and behaviour of structural members, allows for a reduction in the combined wind loads on a structure. The simultaneous load factors are incorporated into 20 recommended load cases that will ensure efficient use of material, and that key structural resisting elements are not overstressed.

4.6.2. Centre Block

Wind load distribution on the Centre Block is governed by the mean (static) wind loads combined with fluctuating wind pressure distributed over the Centre Block, whereas the effects of inertial loads created by building's resonance to the wind excitation are ignored since they are insignificant. The maximum load distributions of the wind loads are evaluated in consideration of the correlation of the fluctuating pressures over the whole structure based on the Load Response Correlation method ¹(LRC).

Three effective static loads which create the 50-year return period peak base shears and torque at the base of the Centre Block are evaluated based on the LRC method, and presented in Figures 9a through 11b in the form of pressure block diagrams. Each effective static load also includes the effects from the jointly acting other directional wind loads. The effective loads illustrated in Figures 9a through 11b should be applied based on the load combinations summarized in Table 5 to account for both opposite directional wind loads.

It is noteworthy that the wind loads illustrated in the Figures do not include the dynamic wind effects (wind instability effects such as vortex shedding) on slender structural components, such as the small towers at the northwest and northeast corners of the Centre Block. These effects are considered minor and do not contribute measurably to the overall wind loads provided.

4.7. Variation of Loads with Future Development

Based on the fact that the Parliamentary Precinct is well established and no building additions or demolitions are expected within the lifetime of the study buildings or nearby neighbouring Parliament buildings, the wind loads determined in this report are expected to remain applicable for the entire design life of the Parliamentary buildings.

¹ M. Kasperski and H.-J. Niemann, "The L.R.C. (load-response-correlation) – method, a general method of estimating unfavourable wind load distribution for linear and non-linear structural behavior," *Journal of Wind Engineering and Industrial Aerodynamics*, 41-44 (1992) 1753-1763.

5. RESULTS AND RECOMMENDATIONS

5.1. Base Shear Forces and Moments

Tables 2a and 2b present the recommended 50-year return period base shear forces, overturning moments and torques for the strength design of the Peace Tower and Centre Block respectively, obtained from measurements and consideration of the Ottawa wind climate. Table 2a also compares the wind tunnel test results with the code values. These load quantities are interpreted according to the axes illustrated in Figures 2a and 2b. It is noteworthy that the axis system in the model is located at the centre of the plan, however, it is acceptable to apply the measured wind forces at other locations in the structure model, such as the centre of rotation, with proper consideration of axis translation or rotation of forces and moments. Figures 4 to 8 illustrate the variation of base moments, shear forces and torque on the Peace Tower evaluated from the analysis described in Section 4.4, using the wind load measured in the wind tunnel as a function of wind direction, and from which the recommended values are derived.

5.2. Wind Load Distribution

Table 4 tabulates the distributions of recommended wind loads on the Peace Tower, taken from Table 2a, on a floor-by-floor basis in both sway directions. For the given building plan, floor-by-floor torque can be obtained simply by offsetting the sway forces from the centre of rotation by an amount sufficient to generate the total base torques. Colour separations have been used to improve the readability of the Tables, and are not related to the interpretation of the conditions.

Figures 9a through 11b represent three load cases of effective static loads on Centre Block, which represent wind load distributions creating 50-year return period maximum base shears and torques.

5.3. Evaluation of Accelerations

Predicted peak accelerations of the Peace Tower are presented in Table 3(a) for the one-year and ten-year return periods and structural damping ratios of 0.015 and 0.020 (i.e. 1.5% and 2.0% of critical damping) at the top occupied floor, corresponding to a height of 86.7 m above grade and a distance of 7 m from the coordinate of origin illustrated in Figure 2a. The peak ten-year acceleration at this floor level is predicted to be approximately 12.3 milli-g (1.2% of gravity) for 0.020 structural damping ratio. The peak torsional velocity for the case, summarized in Table 3(b), is 0.59 milli-rads/sec corresponding to ten-year return periods for structural damping ratio of 0.020.

The International Standards Organization (ISO)² provides guidelines for occupant comfort in buildings with natural frequencies less than 1 Hertz. Interpretation of this and other documents suggests that a suitable peak acceleration limit for residential buildings would be 1.5% to 1.8% of gravity for a ten-year return period (i.e. 15 to 18 milli-g). Office buildings, hotels, and buildings not occupied frequently, can tolerate a slightly higher acceleration limit of 2.0% of gravity. For torsional response, the Council on Tall Buildings and Urban Habitat (CTBUH) provides a provisional guideline limiting torsional velocity at the highest level to 1.5 and 3.0 milli-rads/sec for one-year and ten-year return period winds (Table 3(b)). Higher levels will cause a greater proportion of occupants to complain.

The accelerations are expected to be acceptable based on estimated damping ratio of 2.0%. Torsional velocity is well acceptable based on the noted industry guideline.

5.4. Simultaneous Loads in Orthogonal Directions

To determine the simultaneous load effects on individual structural members, as discussed in Section 4.6 of this report, it is required to consider the recommended load combinations defined in Tables 5 and 6 for the Peace Tower the Centre Block respectively. Colour separations have been used to improve the readability of the Tables, and are not related to the interpretation of the conditions. For structural design purposes, the combination of effective static wind loads shall be applied at each floor level, along with the usual safety factors required by the OBC 2012.

All results in this report represent specified loads, which shall be increased by the usual safety factors as required by the Ontario Building Code (OBC 2012) and structural design standards.

² International Standards Organization (ISO) 2631-3 *Evaluation of Human Exposure to Whole-Body Vibration Part 3: Evaluation of exposure to whole-body z-axis vertical vibration in the frequency range .01 to 0.63Hz* (ISO 1985)
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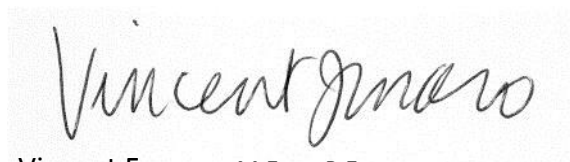
This concludes our assessment and report. If you have any questions or wish to discuss our findings please advise us. In the meantime, we thank you for the opportunity to be of service.

Yours truly,

Gradient Wind Engineering Inc.

A blue ink signature of Un Yong Jeong, written in a cursive style.

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A black ink signature of Vincent Ferraro, written in a cursive style.

Vincent Ferraro, M.Eng., P.Eng.
Principal

Table 1: Building Floor Mass of Peace Tower

Level	Mass (kg×10 ³)	Level	Mass (kg×10 ³)
Level R	6.776	Level F	571.734
Level Q	11.022	Level E	203.588
Level P	13.432	Level D	550.608
Level O	14.330	Level C	385.652
Level N	24.291	Level B	325.252
Level M	18.375	Level A	399.012
Level L	34.361	Refuge Floor	477.448
Level K	30.846	6 th Floor	492.834
Level J	148.394	5 th Floor	446.850
Level I	122.531	4 th Floor	460.729
Level H	621.454	Memorial Chamber	928.352
Level G	329.802	2 nd Floor	489.138
Level F1	382.303		

Note: Mass Moment of Inertia (MMI) data are estimated by using the mass properties based on our experience.

Table 2a: Peace Tower - Predicted 50-Year Return Period¹ Peak Base Moments and Shear Forces², Structural Damping Ratio = 0.020
 $f_x = 1.081$ Hz, $f_y = 1.336$ Hz, $f_z = 2.292$ Hz

	Building Base Moments ($\times 10^5$ kN-m)			Base Shears ($\times 10^3$ kN)	
	Mx	My	Mz	Fx	Fy
Wind Tunnel Test	0.43	0.44	0.0098	0.96	0.91
Code ³	0.59	0.60	-	1.17	1.16

Notes:

1. Results in the table represent specified loads, which should be used with appropriate load factors as specified in OBC 2012.
2. See Figure 2 for the coordinate system axes.
3. Code values correspond to the 80% of Exposure B in OBC 2012.

Table 2b: Centre Block - Predicted 50-Year Return Period Mean (Static) Base Shear Forces

	Mean (Static) Base Torque ($\times 10^5$ kN-m)	Mean (Static) Base Shears ($\times 10^3$ kN)	
	Mz	Fx	Fy
Wind Tunnel Test	0.42	1.3	1.9
Code ¹	-	1.3	2.4

Notes:

1. Code values are estimated considering the effects of the topography, which is expected to be important for the Centre Block.

**Table 3(a): Peace Tower - Predicted Peak Resultant Accelerations¹ ($g \times 10^{-3}$)²
at the Top Continuously Occupied Level
 $f_x = 1.081$ Hz, $f_y = 1.336$ Hz, $f_z = 2.292$ Hz**

Damping Ratio	Return Period	X – Sway	Y – Sway	Torsion ³	Resultant ⁴	ISO ⁵ Criteria
0.015	1 year	5.1	5.7	4.6	7.8	12.0
	10 year	10.7	9.2	7.8	14.2	20.0
0.020	1 year	4.4	4.9	4.0	6.0	12.0
	10 year	9.3	8.0	6.8	12.3	20.0

Notes:

1. Acceleration calculated at the top continuously occupied level.
2. $g \times 10^{-3}$ equivalent to milli-g.
3. Torsion calculated at approximately 7 m from the coordinate origin.
4. Peak values determined from root-mean-square multiplied by peak factor.
5. International Standards Organization (ISO) 6897-1984: Guidelines for the evaluation of the response of occupants of fixed structures, especially buildings and off-shore structures, to low-frequency horizontal motion (0.063 to 1 Hertz).

Table 3(b): Peace Tower - Comparison of Torsional Response

Damping Ratio	Return Period	Torsional Acceleration ¹ milli-g	Torsional Velocity ² milli-rads/sec	CTBUH ³ Criteria milli-rads/sec
0.015	1 year	4.6	0.40	1.5
	10 year	7.8	0.68	3.0
0.020	1 year	4.0	0.35	1.5
	10 year	6.8	0.59	3.0

Notes:

1. Torsion calculated at approximately 7 m from the coordinate origin at the top occupied floor.
2. Peak values determined from root-mean-square multiplied by peak factor.
3. Council on Tall Buildings and Urban Habitat (CTBUH), Chicago, IL.

Table 4: Estimated Effective Static Wind Loads on Peace Tower at Each Level
Corresponding to 50-Year Base Moments
Structural Damping of 0.020
 $f_x = 1.081 \text{ Hz}$, $f_y = 1.336 \text{ Hz}$, $f_z = 2.292 \text{ Hz}$

Level (elevation)	Height m	F _x kN	F _y kN	M _z kN-m
Level R (90.58)	88.71	7.1	22.3	1.6
Level Q (88.60)	86.73	16.7	29.3	2.5
Level P (86.04)	84.16	18.1	29.1	3.1
Level O (83.48)	81.60	19.3	28.8	3.3
Level N (80.92)	79.03	21.6	28.8	5.5
Level M (78.36)	76.47	21.8	28.2	4.2
Level L (75.80)	73.90	24.6	28.3	7.8
Level K (73.24)	71.34	39.7	41.2	6.9
Level J (69.43)	67.53	63.0	53.7	33.3
Level I (64.53)	62.63	55.6	47.9	27.4
Level H (59.90)	58.00	95.3	58.7	132.3
Level G (54.25)	52.35	50.1	31.5	69.9
Level F1 (54.25)	48.98	47.6	31.4	80.5
Level F (47.58)	45.68	70.5	49.7	116.6
Level E (42.07)	40.17	15.0	8.2	40.8
Level D (38.56)	39.24	64.1	48.4	107.4
Level C (35.27)	33.37	33.0	30.6	62.2
Level B (31.98)	30.08	33.4	30.9	42.7
Level A (28.69)	26.79	37.3	36.1	42.6
Refuge Floor (24.89)	22.99	24.7	33.0	44.6
6 th Floor (22.93)	21.03	37.6	44.0	42.1
5 th Floor (19.12)	17.22	37.0	39.8	31.3
4 th Floor (15.31)	13.41	36.3	37.2	25.2
Memorial Chamber Floor (11.63)	9.73	50.9	64.6	36.9
2 nd Floor (5.71)	3.81	39.6	28.6	9.5
Σ		960.0	910.0	980.0

Table 5: Load Combinations for Peace Tower

Load Case	X-Axis (%)	Y-Axis (%)	Z-Axis (%)
1	+100	+40	+40
2	+100	+40	-40
3	+100	-40	+40
4	+100	-40	-40
5	-100	+40	+40
6	-100	+40	-40
7	-100	-40	+40
8	-100	-40	-40
9	+40	+100	+40
10	+40	+100	-40
11	-40	+100	+40
12	-40	+100	-40
13	+40	-100	+40
14	+40	-100	-40
15	-40	-100	+40
16	-40	-100	-40
17	+50	+50	+100
18	+50	-50	+100
19	-50	+50	-100
20	-50	-50	-100

Notes:

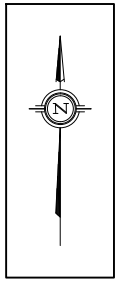
1. The above load combinations apply to the effective static wind loads on peace tower tabulated in Table 3;
2. 'X-Axis (%)' refers to Fx forces and My moments;
3. 'Y-Axis (%)' refers to Fy forces and Mx moments;
4. 'Z-Axis (%)' refers to torsion, Mz;

Table 6: Load Combinations for Centre Block

Load Combination	Case 1 (%)	Case 2 (%)	Case 3 (%)
1	+100	0	0
2	-100	0	0
3	0	+100	0
4	0	-100	0
5	0	0	+100
6	0	0	-100

Notes:

1. The above load combination applies to the effective static wind loads on the Centre Block illustrated in Figures 9a to 11b.
2. Case 1 represents the wind condition creating the highest moment, M_x , and shear force, F_y ;
3. Case 2 represents the wind condition creating the highest moment M_y and shear force, F_x ;
4. Case 3 represents the wind condition creating the highest torque, M_z ;
5. Load cases 1 to 3 include the effects of concurrently acting other directional wind load components.

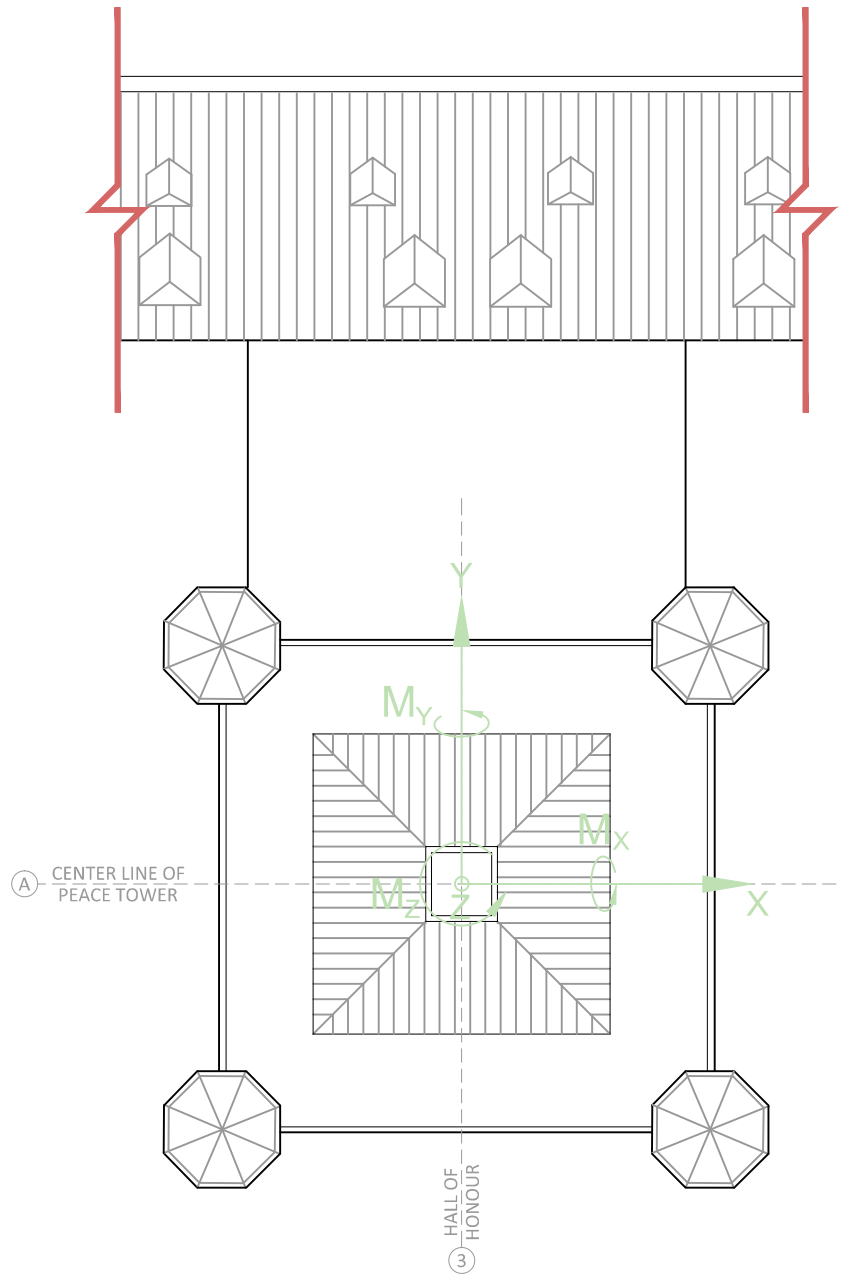
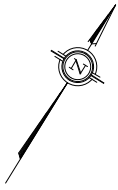


OTTAWA RIVER

RIDEAU CANAL

WELLINGTON STREET

METCALFE STREET

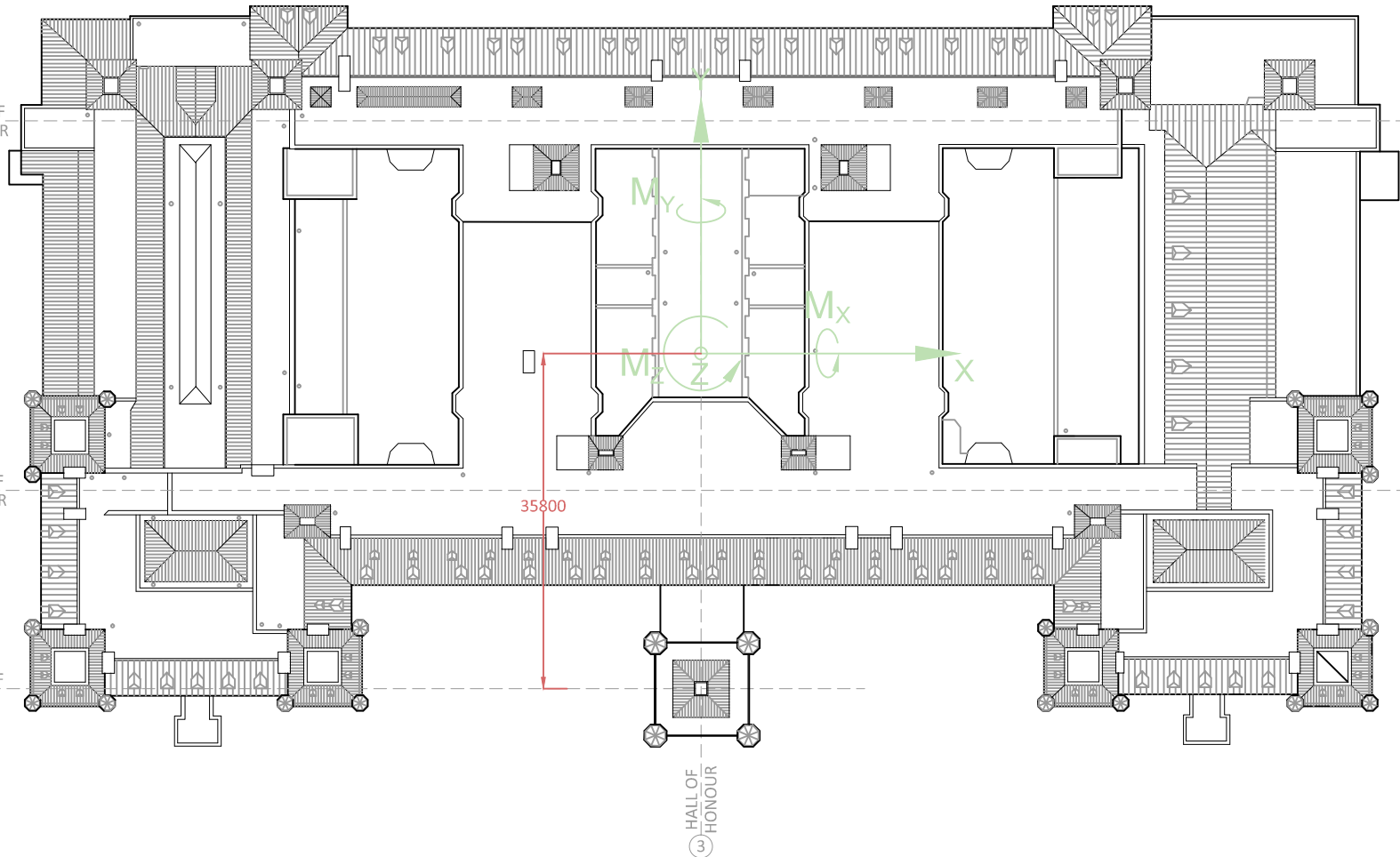


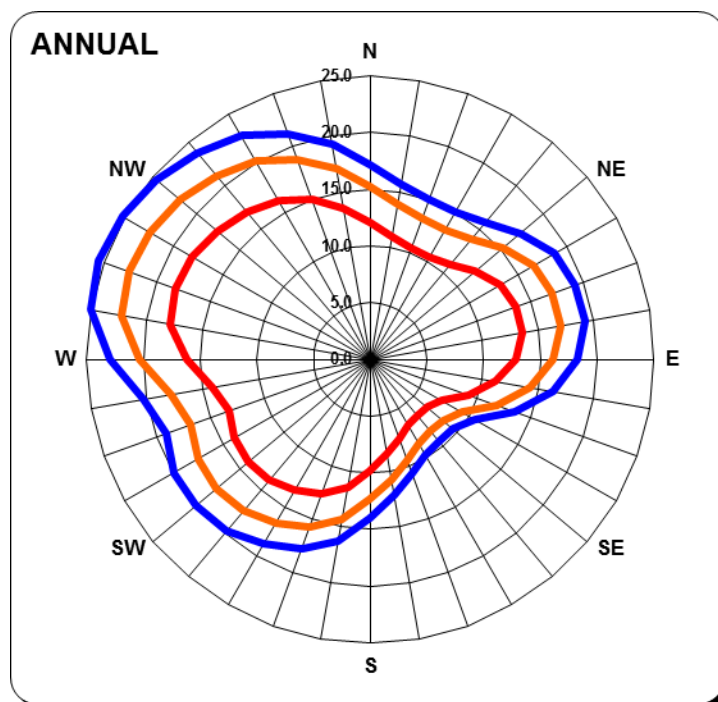


③ CENTER LINE OF NORTH CORRIDOR

② CENTER LINE OF SOUTH CORRIDOR

① CENTER LINE OF PEACE TOWER



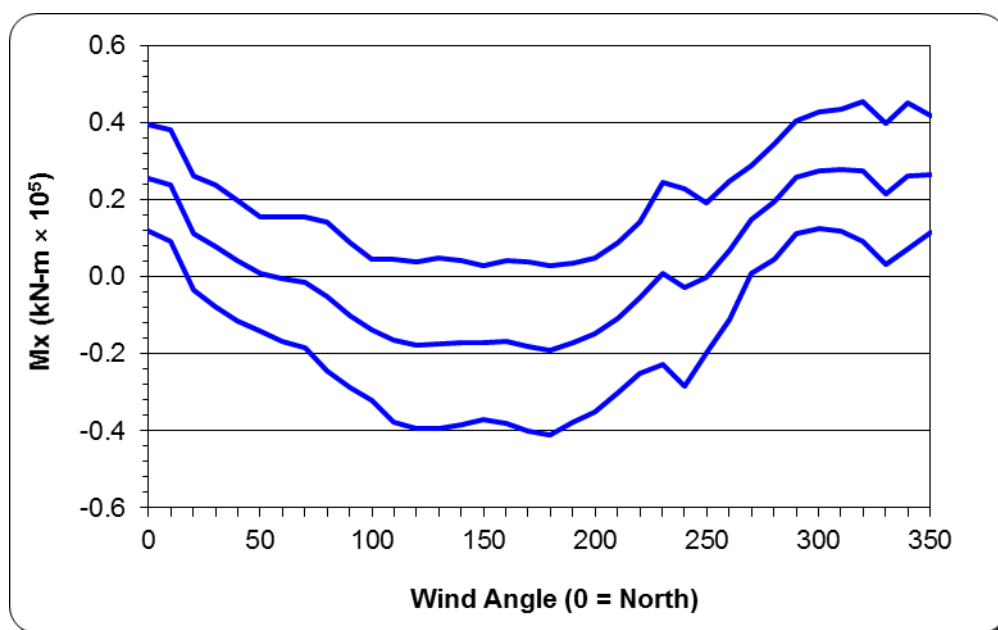


ANNUAL

Note:

1. Radial distances indicate wind speed in meters per second (m/s) at 10 m above grade.
2. A point along the innermost contour represents the wind speed exceeded on average 0.01% (once per year) of the time within a 10° sector centered on that direction. The middle and outermost contours represent probability levels of 0.001% (once per 10 years) and 0.00023% (once every 50 years), respectively.

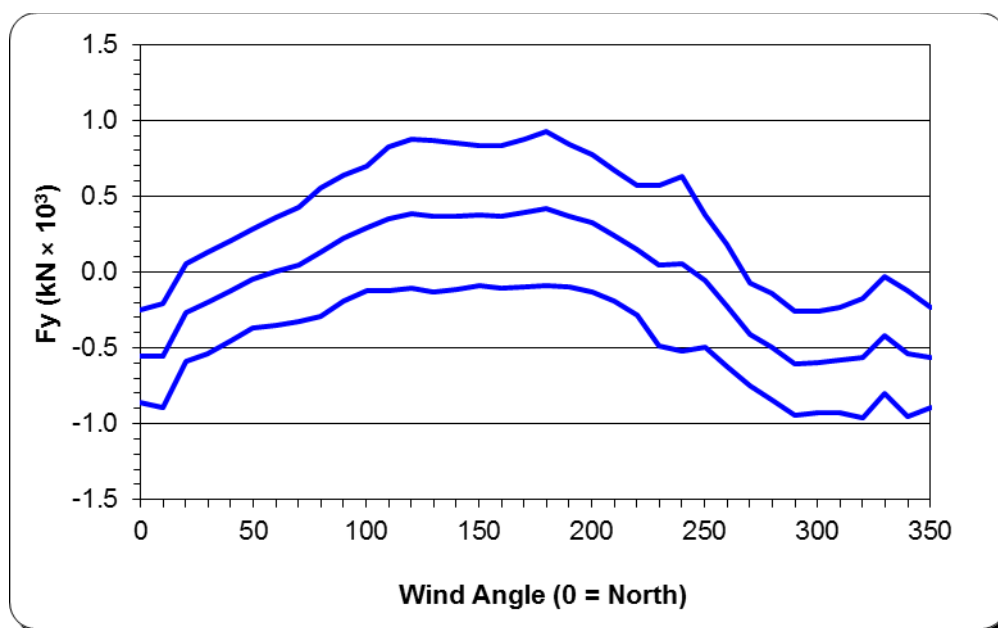
**Figure 3: Annual Distribution of Wind for Various Probability Levels,
Ottawa International Airport, Ottawa, Ontario**



**Figure 4: Base Bending Moments M_x versus Wind Direction
from Wind Tunnel Measurements, Structural Damping Ratio = 0.020**

$f_x = 1.081 \text{ Hz}$, $f_y = 1.336 \text{ Hz}$, $f_z = 2.292 \text{ Hz}$

(Top Curve = Max, Middle Curve = Mean, Bottom Curve = Min)



**Figure 5: Base Shear Forces F_y versus Wind Direction
from Wind Tunnel Measurements, Structural Damping Ratio = 0.020**

$f_x = 1.081 \text{ Hz}$, $f_y = 1.336 \text{ Hz}$, $f_z = 2.292 \text{ Hz}$

(Top Curve = Max, Middle Curve = Mean, Bottom Curve = Min)

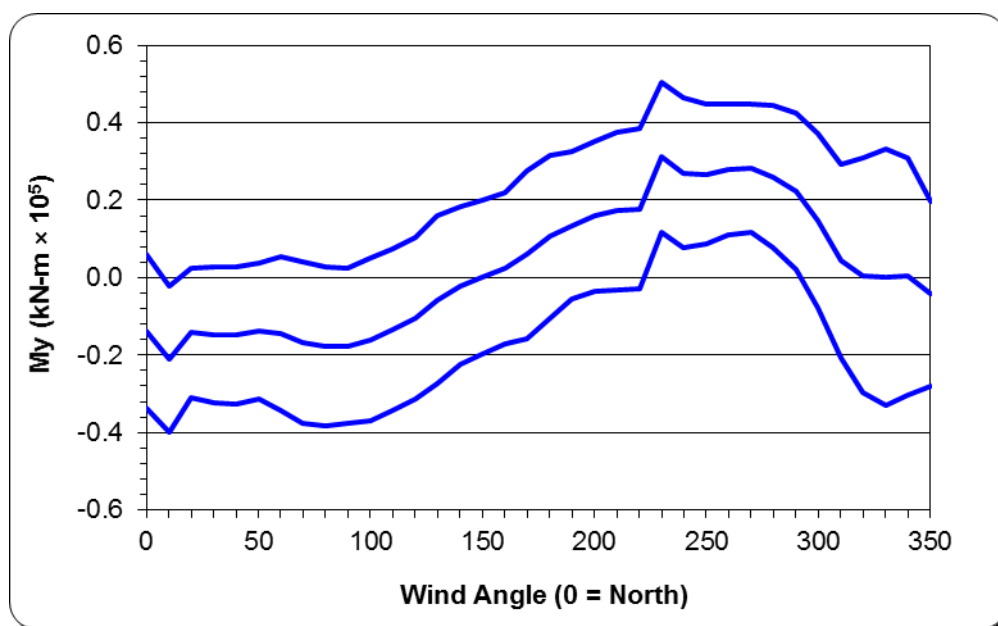


Figure 6: Base Bending Moments M_y versus Wind Direction
from Wind Tunnel Measurements, Structural Damping Ratio = 0.020
 $f_x = 1.081$ Hz, $f_y = 1.336$ Hz, $f_z = 2.292$ Hz
 (Top Curve = Max, Middle Curve = Mean, Bottom Curve = Min)

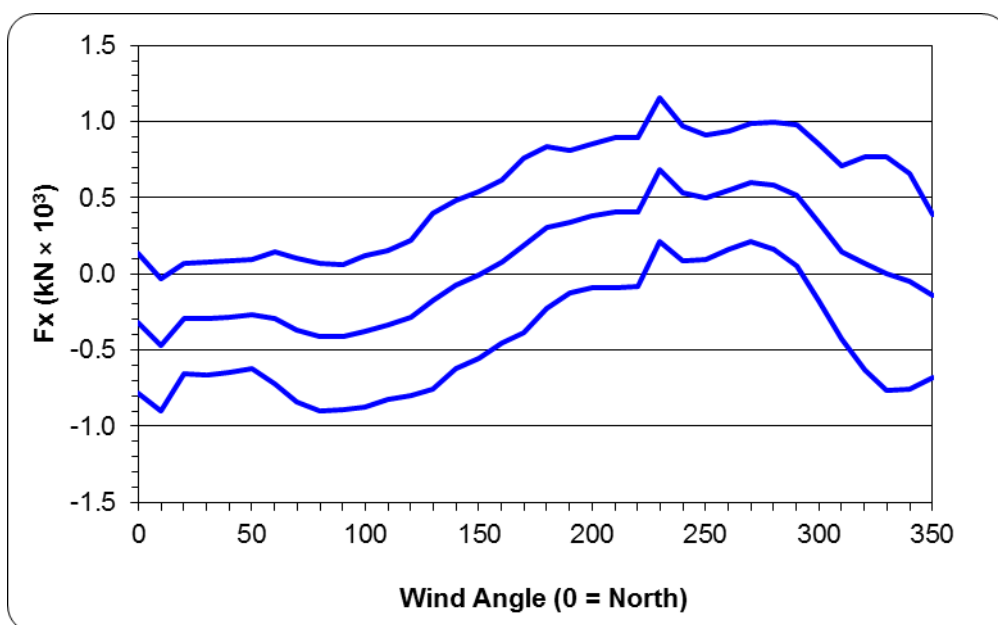


Figure 7: Base Shear Forces F_x versus Wind Direction
from Wind Tunnel Measurements, Structural Damping Ratio = 0.020
 $f_x = 1.081$ Hz, $f_y = 1.336$ Hz, $f_z = 2.292$ Hz
 (Top Curve = Max, Middle Curve = Mean, Bottom Curve = Min)

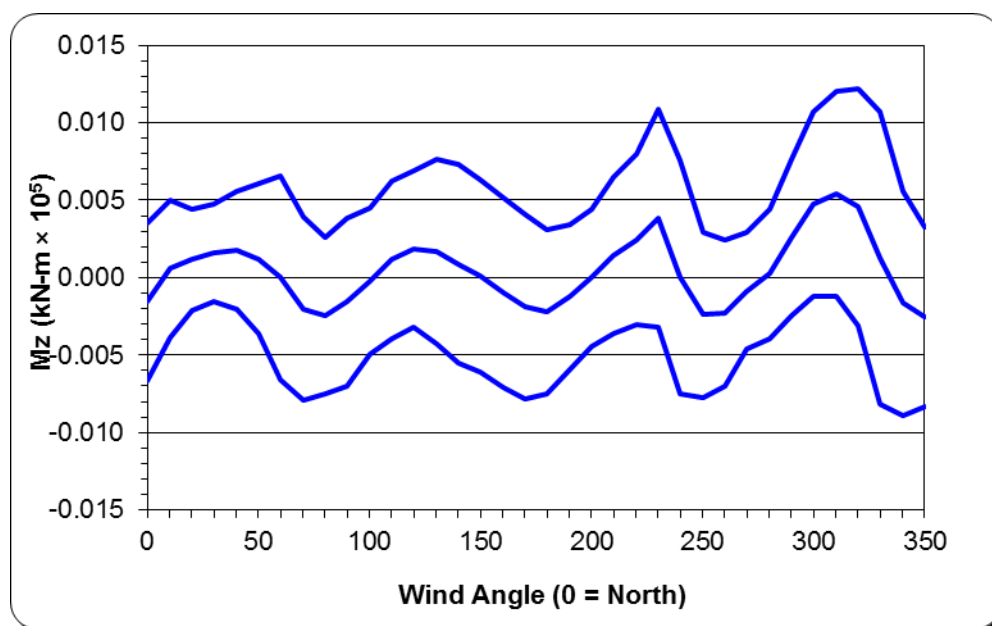
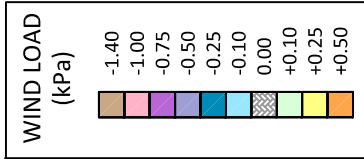


Figure 8: Base Torsional Moments M_z versus Wind Direction from Wind Tunnel Measurements, Structural Damping Ratio = 0.020

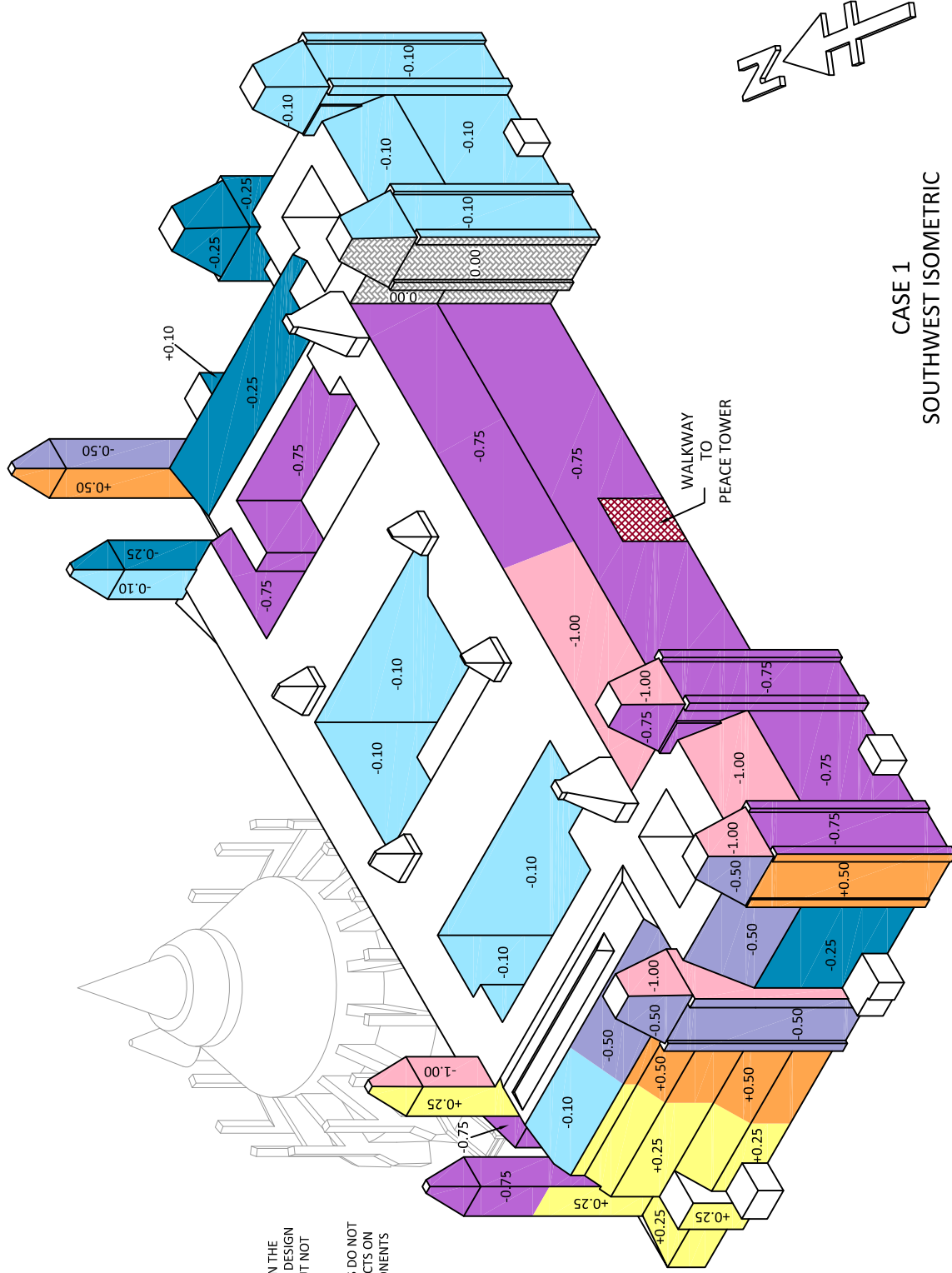
$f_x = 1.081$ Hz, $f_y = 1.336$ Hz, $f_z = 2.292$ Hz

(Top Curve = Max, Middle Curve = Mean, Bottom Curve = Min)




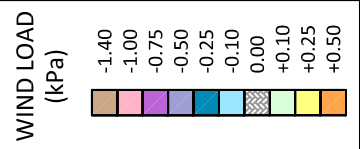
NOTES:

1. WIND LOADS ILLUSTRATED IN THE DRAWING ONLY APPLY TO THE DESIGN OF THE MAIN STRUCTURES, BUT NOT APPLICABLE TO THE CLADDING
2. THE SPECIFIED WIND LOADS DO NOT INCLUDE DYNAMIC WIND EFFECTS ON SLENDER STRUCTURAL COMPONENTS SUCH AS SMALL TOWERS.



CASE 1
SOUTHWEST ISOMETRIC

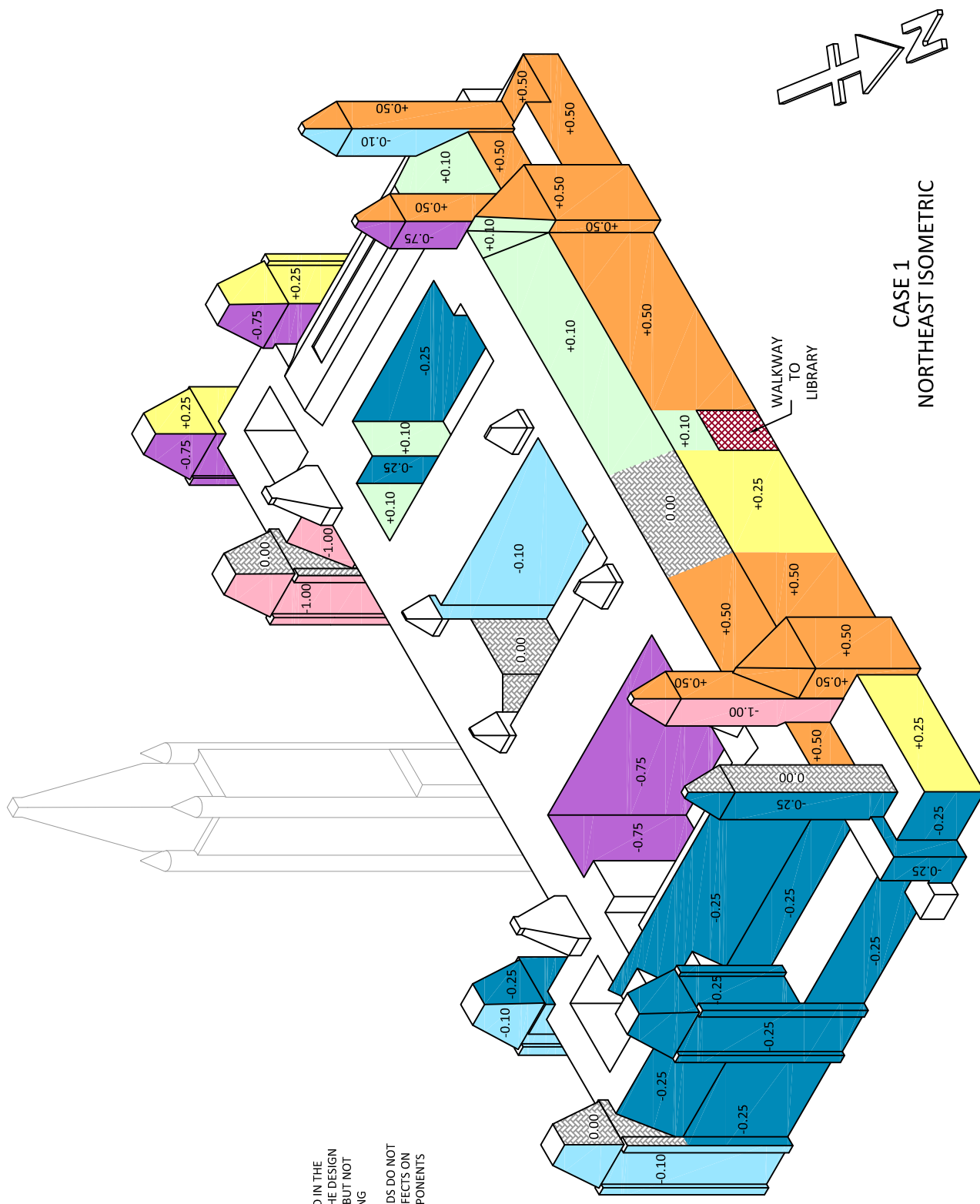
<div><div>127 Walgreen Road Ottawa, Ontario (613) 836 0934</div><div>GRADIENT WIND ENGINEERING INC</div></div>		<div>PROJECT</div> <div>CENTER BLOCK, OTTAWA ONTARIO</div> <div>STRUCTURAL WIND LOAD AND BUILDING MOTION STUDY</div> <div>SCALE1:800 (APPROX)</div> <div>DATEMAY 20, 2015</div>		<div>DESCRIPTION</div> <div>FIGURE 9A: CASE 1</div> <div>EFFECTIVE STATIC WIND LOADS FOR THE STRUCTURAL DESIGN OF CENTER BLOCK BASED ON A FIFTY YEAR RETURN PERIOD.</div>	
		<div>DRAWING NO. GWE15-032-HFPI-9A</div> <div>DRAWN BY B.J.</div>			



NOTES:

1. WIND LOADS ILLUSTRATED IN THE DRAWING ONLY APPLY TO THE DESIGN OF THE MAIN STRUCTURES, BUT NOT APPLICABLE TO THE CLADDING

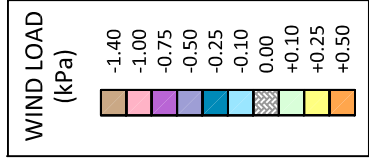
2. THE SPECIFIED WIND LOADS DO NOT INCLUDE DYNAMIC WIND EFFECTS ON SLENDER STRUCTURAL COMPONENTS SUCH AS SMALL TOWERS.



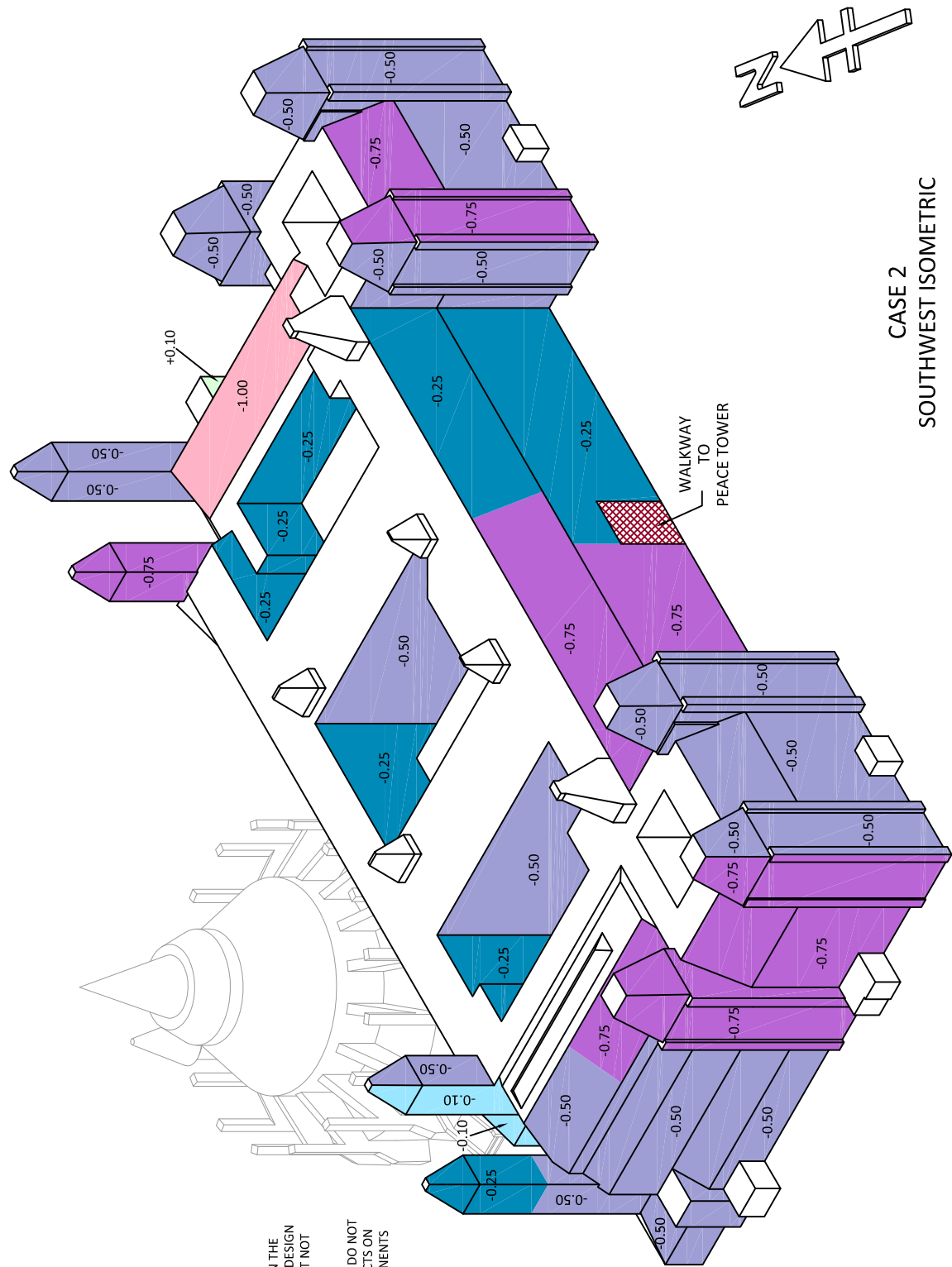
CASE 1
NORTHEAST ISOMETRIC

<div> <div>127 Walgreen Road Ottawa, Ontario (613) 836 0934</div> <div> <div>GW</div> <div>E</div> </div> <div>GRADIENTWIND ENGINEERING INC</div> </div>	PROJECT		CENTER BLOCK, OTTAWA ONTARIO		DESCRIPTION	
	SCALE		1:800 (APPROX)		STRUCTURAL WIND LOAD AND BUILDING MOTION STUDY	
	DATE		MAY 20, 2015		DRAWN BY B.J.	


FIGURE 9B: CASE 1
EFFECTIVE STATIC WIND LOADS FOR THE STRUCTURAL DESIGN
OF CENTER BLOCK BASED ON A FIFTY YEAR RETURN PERIOD.

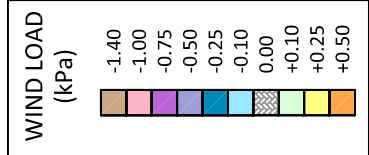


- NOTES:**
1. WIND LOADS ILLUSTRATED IN THE DRAWING ONLY APPLY TO THE DESIGN OF THE MAIN STRUCTURES, BUT NOT APPLICABLE TO THE CLADDING
 2. THE SPECIFIED WIND LOADS DO NOT INCLUDE DYNAMIC WIND EFFECTS ON SLENDER STRUCTURAL COMPONENTS SUCH AS SMALL TOWERS.

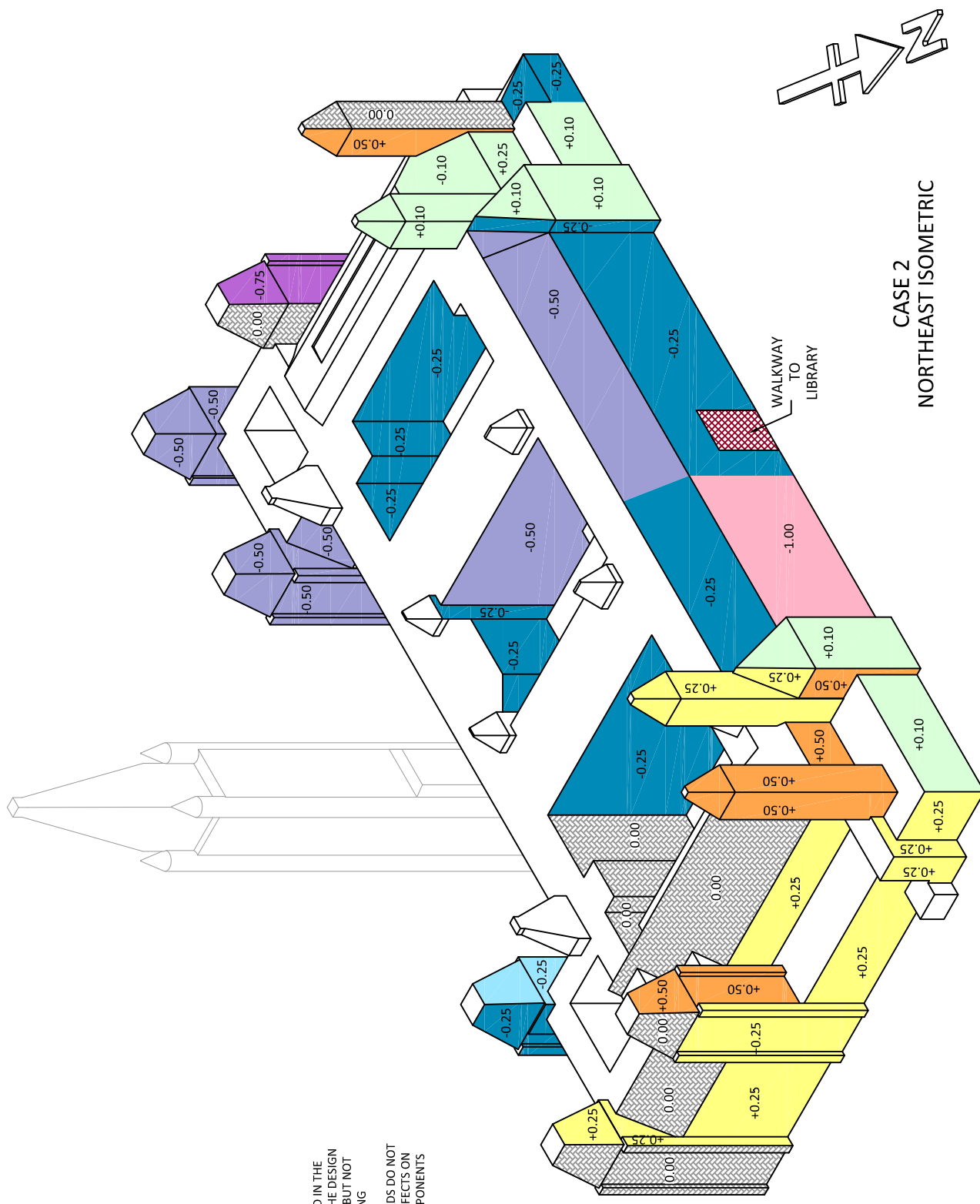


CASE 2
SOUTHWEST ISOMETRIC


<div><p>127 Walgreen Road Ottawa, Ontario (613) 836 0934</p></div>	PROJECT		CENTER BLOCK, OTTAWA ONTARIO		DESCRIPTION FIGURE 10A: CASE 2 EFFECTIVE STATIC WIND LOADS FOR THE STRUCTURAL DESIGN OF CENTER BLOCK BASED ON A FIFTY YEAR RETURN PERIOD.
	SCALE		STRUCTURAL WIND LOAD AND BUILDING MOTION STUDY		
	1:800		DRAWING NO. GWE15-032-HFPI-10A		
	(APPROX.)				
	DATE		DRAWN BY		
	MAY 20, 2015		B.J.		

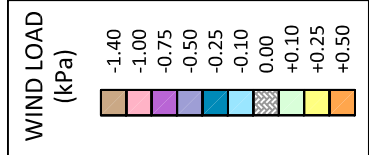


- NOTES:**
1. WIND LOADS ILLUSTRATED IN THE DRAWING ONLY APPLY TO THE DESIGN OF THE MAIN STRUCTURES, BUT NOT APPLICABLE TO THE CLADDING
 2. THE SPECIFIED WIND LOADS DO NOT INCLUDE DYNAMIC WIND EFFECTS ON SLENDER STRUCTURAL COMPONENTS SUCH AS SMALL TOWERS.



**CASE 2
NORTHEAST ISOMETRIC**

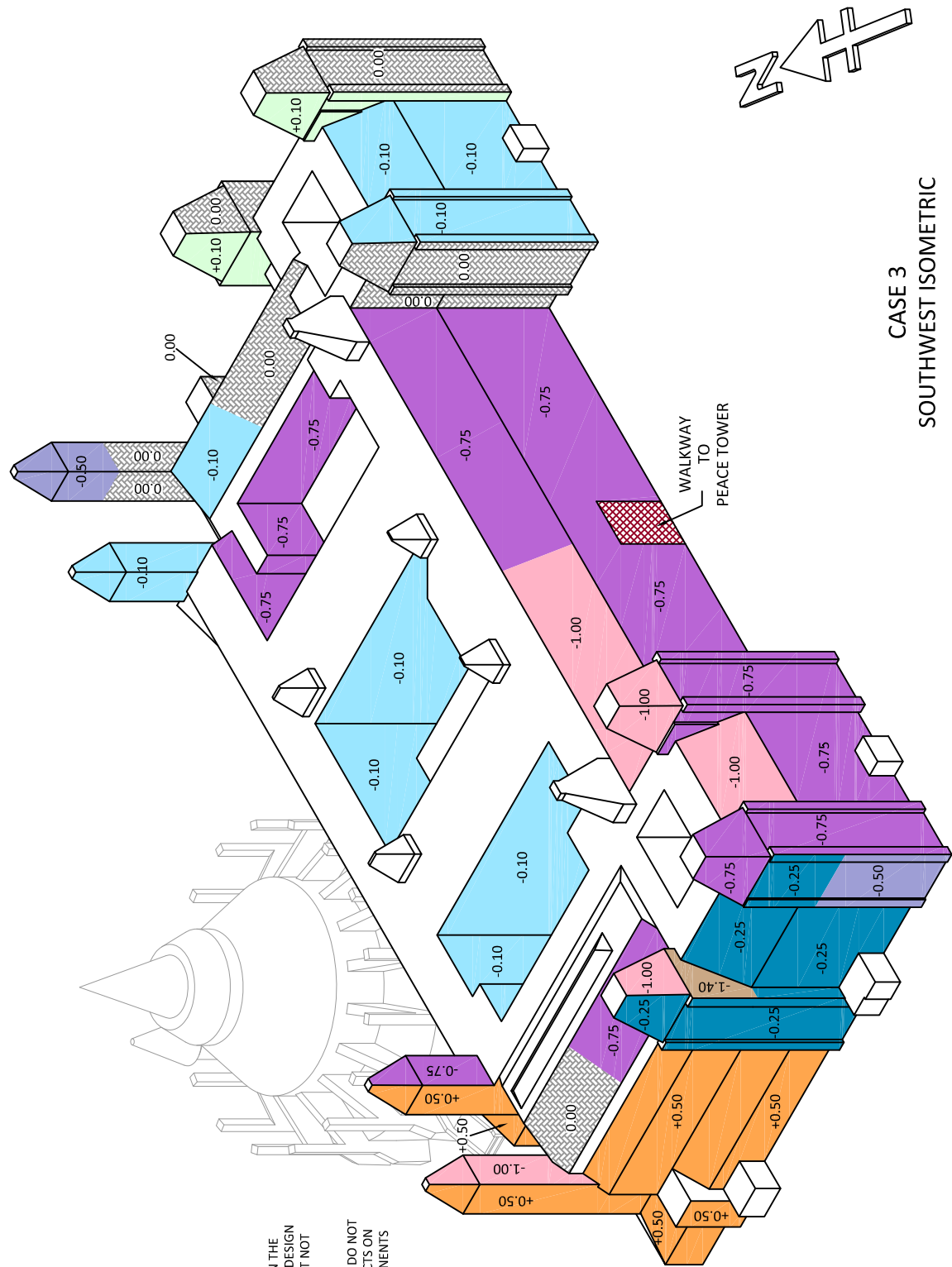
 <p>127 Walgreen Road Ottawa, Ontario (613) 836 0934</p>	<p>PROJECT: CENTER BLOCK, OTTAWA ONTARIO</p>		<p>DESCRIPTION</p> <p>FIGURE 10B: CASE 2</p> <p>EFFECTIVE STATIC WIND LOADS FOR THE STRUCTURAL DESIGN OF CENTER BLOCK BASED ON A FIFTY YEAR RETURN PERIOD.</p>
	<p>SCALE: 1:800 (APPROX)</p>	<p>DRAWING NO: GWE15-032-HFPI-10B</p>	
	<p>DATE: MAY 20, 2015</p>	<p>DRAWN BY: B.J.</p>	




NOTES:

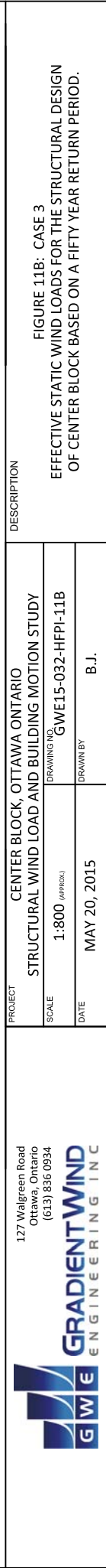
1. WIND LOADS ILLUSTRATED IN THE DRAWING ONLY APPLY TO THE DESIGN OF THE MAIN STRUCTURES, BUT NOT APPLICABLE TO THE CLADDING

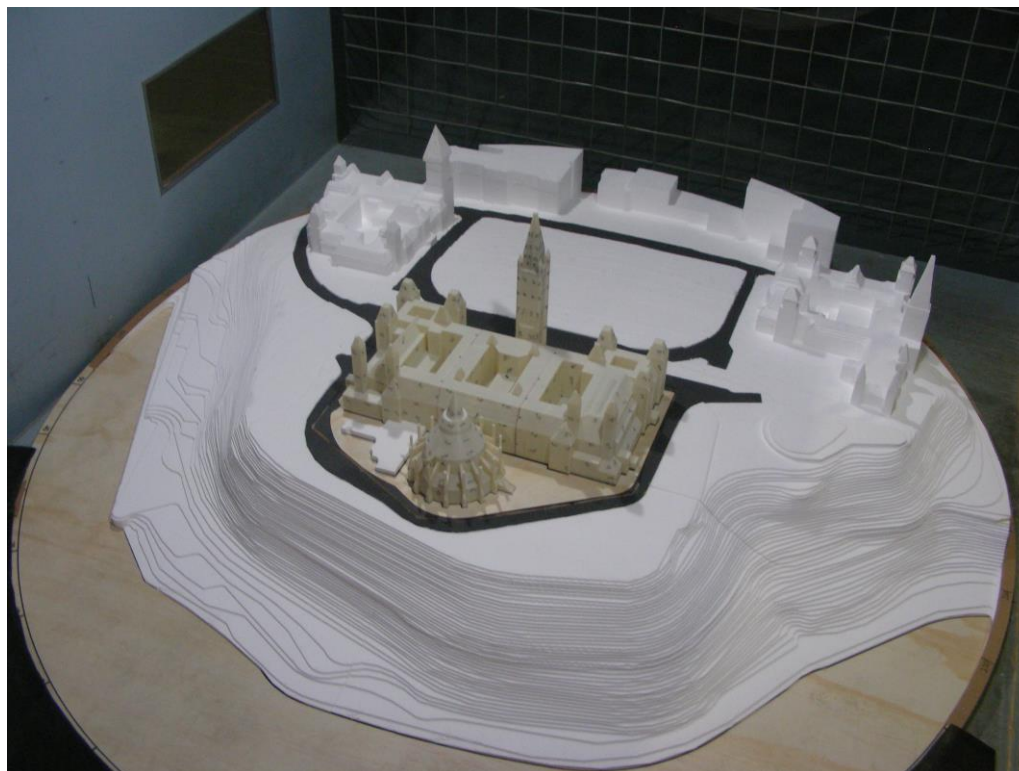
2. THE SPECIFIED WIND LOADS DO NOT INCLUDE DYNAMIC WIND EFFECTS ON SLENDER STRUCTURAL COMPONENTS SUCH AS SMALL TOWERS.



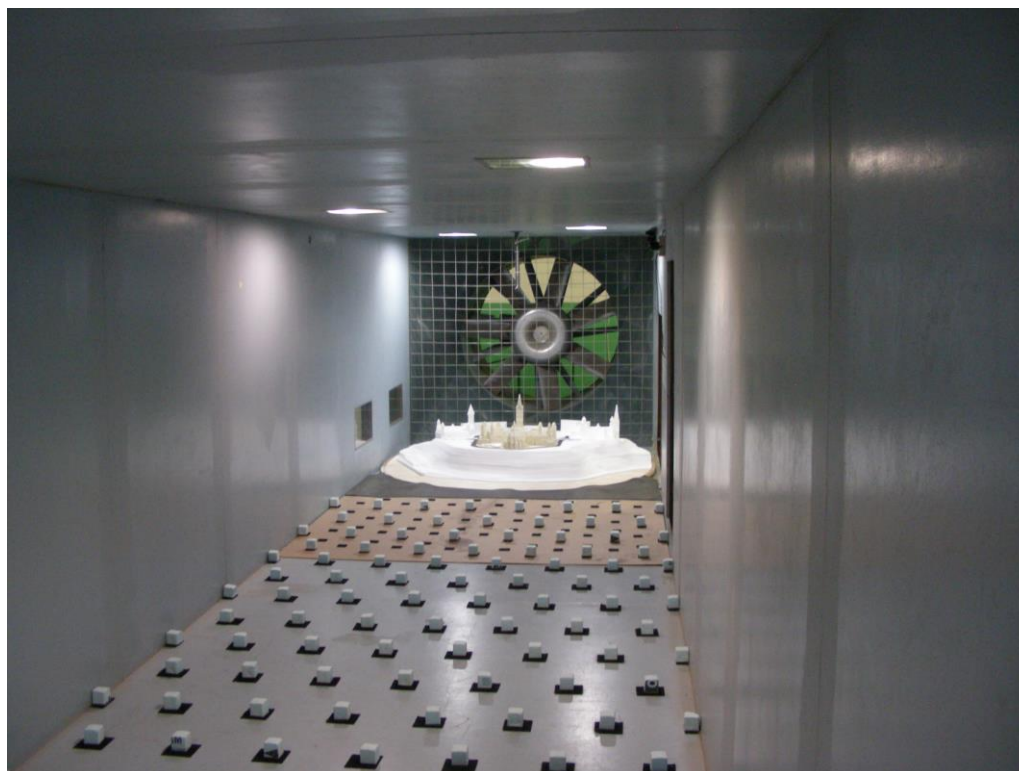
CASE 3
SOUTHWEST ISOMETRIC

 127 Walgreen Road Ottawa, Ontario (613) 836 0934		PROJECT		CENTER BLOCK, OTTAWA ONTARIO		DESCRIPTION FIGURE 11A: CASE 3 EFFECTIVE STATIC WIND LOADS FOR THE STRUCTURAL DESIGN OF CENTER BLOCK BASED ON A FIFTY YEAR RETURN PERIOD.
		SCALE		STRUCTURAL WIND LOAD AND BUILDING MOTION STUDY		
		1:800		DRAWING NO. GWE15-032-HFPI-11A		
		(APPROX.)				
		DATE		MAY 20, 2015		
				DRAWN BY B.J.		





PHOTOGRAPH 1: 1:250 SCALE MODEL INSTALLED IN GWE WIND



PHOTOGRAPH 2: 1:250 SCALE MODEL IN GWE WIND TUNNEL, LOOKING DOWNWIND



PHOTOGRAPH 3: 1:250 SCALE MODEL IN GWE WIND TUNNEL, LOOKING UPWIND



PHOTOGRAPH 4: DETAILED VIEW OF PRESSURE INTEGRATION MODEL

APPENDIX A

WIND TUNNEL SIMULATION OF THE NATURAL WIND

WIND TUNNEL SIMULATION OF THE NATURAL WIND

Wind flowing over the surface of the earth develops a boundary layer due to the drag produced by surface features such as vegetation and man-made structures. Within this boundary layer the mean wind speed varies from zero at the surface to the gradient wind speed at the top of the layer. The height of the top of the boundary layer is referred to as the gradient height, above which the velocity remains more-or-less constant for a given synoptic weather system. The mean wind speed is taken to be the average value over one hour. Superimposed on the mean wind speed are fluctuating (or turbulent) components in the longitudinal (i.e. along wind), vertical and lateral directions. Although turbulence varies according to the roughness of the surface, the turbulence level generally increases from nearly zero (smooth flow) at gradient height to maximum values near the ground. While for a calm ocean, the maximum could be 20%, the maximum for a very rough surface such as the centre of a city could be 100%, or equal to the local mean wind speed. The height of the boundary layer varies in time and over different terrain roughness within the range of 400 m to 600 m.

Simulating real wind behaviour in a wind tunnel requires simulating the variation of mean wind speed with height, simulating the turbulence intensity, and matching the typical length scales of turbulence. It is the ratio between wind tunnel turbulence length scales and turbulence scales in the atmosphere that determines the geometric scales that models can assume in a wind tunnel. Hence, when a 1:400 scale model is quoted, this implies that the turbulence scales in the wind tunnel and the atmosphere have the same ratios. Some flexibility in this requirement has been shown to produce reasonable wind tunnel predictions compared to full scale. In model scale the mean and turbulence characteristics of the wind are obtained with the use of spires at one end of the tunnel and roughness elements along the floor of the tunnel. The fan is located at the model end and wind is pulled over the spires, roughness elements and model. It has been found that, to a good approximation, the mean wind profile can be represented by a power law relation, shown below, giving height above ground versus wind speed.

$$U = U_g \left(\frac{Z}{Z_g} \right)^\alpha$$

Where; U = mean wind speed, U_g = gradient wind speed, Z = height above ground, Z_g = depth of the boundary layer (gradient height) and α is the power law exponent.

Figure A1 plots three such profiles for the open country, suburban and urban exposures.

The exponent α varies according to the type of terrain; $\alpha = 0.14, 0.25$ and 0.33 for open country, suburban and urban exposures respectively. Figure A2 illustrates the theoretical variation of turbulence in full scale and some wind tunnel measurement for comparison.

The integral length scale of turbulence can be thought of as an average size of gust in the atmosphere. Although it varies with height and ground roughness it has been found to generally be in the range of 100 m to 200 m in the upper half of the boundary layer. Thus, for a 1:300 scale, the model value should be between 1/3 and 2/3 of a meter. Integral length scales are derived from power spectra, which describe the energy content of wind as a function of frequency. There are several ways of determining integral length scales of turbulence. One way is by comparison of a measured power spectrum in model scale to a non-dimensional theoretical spectrum such as the Davenport spectrum of longitudinal turbulence. Using the Davenport spectrum, which agrees well with full-scale spectra, one can estimate the integral scale by plotting the theoretical spectrum with varying L until it matches as closely as possible the measured spectrum:

$$f \times S(f) = \frac{\frac{4(Lf)^2}{U_{10}^2}}{\left[1 + \frac{4(Lf)^2}{U_{10}^2}\right]^{\frac{4}{3}}}$$

Where, f is frequency, $S(f)$ is the spectrum value at frequency f , U_{10} is the wind speed 10 m above ground level, and L is the characteristic length of turbulence.

Once the wind simulation is correct, the model, constructed to a suitable scale, is installed at the centre of the working section of the wind tunnel. Different wind directions are represented by rotating the model to align with the wind tunnel centre-line axis.

References

1. Teunissen, H.W., 'Characteristics Of The Mean Wind And Turbulence In The Planetary Boundary Layer', Institute For Aerospace Studies, University Of Toronto, UTIAS # 32, Oct. 1970
2. Flay, R.G., Stevenson, D.C., 'Integral Length Scales In An Atmospheric Boundary Layer Near The Ground', 9th Australian Fluid Mechanics Conference, Auckland, Dec. 1966
3. ESDU, 'Characteristics of Atmospheric Turbulence Near the Ground', 74030
4. Bradley, E.F., Coppin, P.A., Katen, P.C., 'Turbulent Wind Structure Above Very Rugged Terrain', 9th Australian Fluid Mechanics Conference, Auckland, Dec. 1966

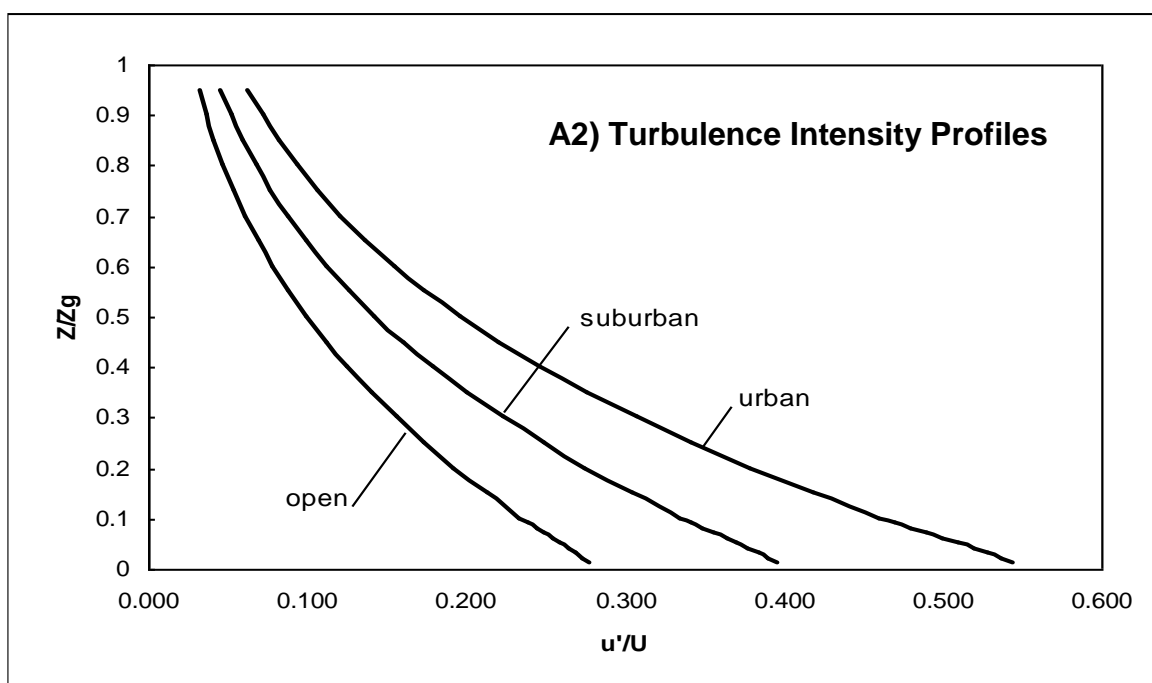
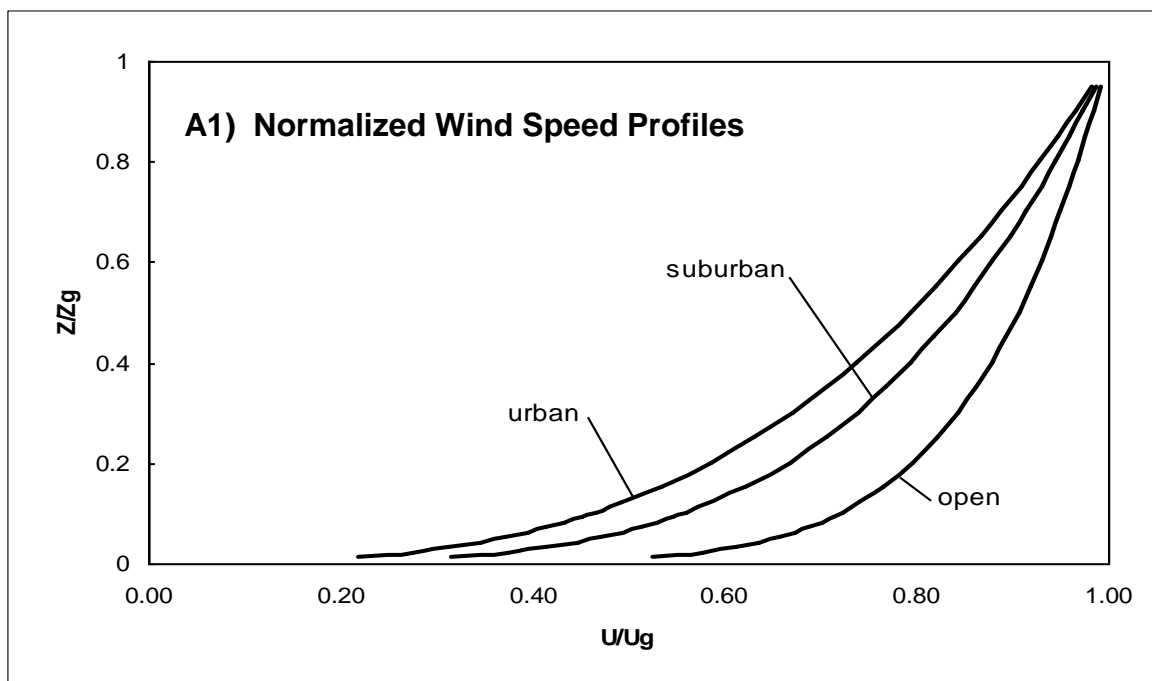


Figure A1 (Top): Mean Wind Speed Profiles

Figure A2 (Bottom): Turbulence Intensity Profiles (u' = fluctuation of mean velocity)

APPENDIX B

THE HIGH-FREQUENCY PRESSURE INTEGRATION METHOD

THE HIGH-FREQUENCY PRESSURE INTEGRATION METHOD

Wind loads on flexible structures produce mean and dynamic responses. The mean loading can be obtained in one of several ways, including a cladding pressure test. The dynamic component can be considered to arise from two distinct effects including, a quasi-steady loading and a fluctuating effect, both of which can be determined with a dynamic model. Whereas the quasi-steady loading arises from the correlation of wind forces over the surface of the building, the fluctuating forces arise as the building mass responds to the applied forces at its natural frequency and mode of vibration. This latter quantity is the resonant component of the response. The high-frequency pressure integration approach is based on modal analysis theory which measures the quasi-steady loading governed by the shape of the building. The resonant response is determined by combining this quasi-steady loading with the mechanical admittance function representing the mass, stiffness and structural damping in each mode. For most tall buildings the majority of the response occurs in the fundamental modes of vibration, one for sway in each principal structural direction, and one in torsion. In addition, the aerodynamic damping for most tall buildings is positive and negligible, and therefore not important at practical wind speeds.

The equation of motion of a building, with height H is defined on the basis of Newton's second law of motion with additional factors to account for stiffness and damping as follows:

$$m_j^* \left(\frac{d^2 q_j}{dt^2} + 2\zeta_j \omega_j^2 \frac{dq_j}{dt} + \omega_j^2 q_j \right) = F_j^* ; \quad (1)$$

$$F_j^* \cong \sum_i [F_{xi} \phi_{xij} + F_{yi} \phi_{yij} + F_{\theta i} \phi_{\theta ij}] ; \quad (2)$$

$$F_{xi}(t) = p_i(t) a_{xi} ; \quad (3)$$

$$F_{yi}(t) = p_i(t) a_{yi} ; \quad (4)$$

$$F_{\theta i}(t) = -F_{xi}(t) \Delta_{yi} + F_{yi}(t) \Delta_{xi} ; \quad (5)$$

where

$$m_j^* = \text{generalized mass} = \int_0^H [m \phi_{xj}^2(z) + m \phi_{yj}^2 + I \phi_{\theta j}^2] dz ;$$

ϕ_{xj} , ϕ_{yj} and $\phi_{\theta j}$ = the generalized coordinate of mode ϕ in x , y and θ direction;

m = mass per unit height;

I = mass moment of inertia per unit height;

ζ_j = structural damping ratio of mode j ;

ω_j = natural frequency of mode j (radians/second);

$p_i(t)$ = instantaneous pressure corresponding to the tributary area i ;

a_{xi} and a_{yi} = areas normal to the x and y directions respectively for the tributary area i ;

$\Delta_{xi} = x_i - x_o$;

$\Delta_{yi} = y_i - y_o$;

x_i and y_i = x and y coordinates of tributary area i ;

x_o and y_o = x and y coordinates of the centre of rotation.

The sway of the building x as a function of the vertical coordinate z is

$$x(z,t) = \sum_j \phi_j(z) q_j(t). \quad (6)$$

Provided that the aerodynamic forces expressed in equations (3) to (5) are independent of building motions q_j then the solution of equation (1) can be derived in terms of the power spectra $S_{q_j}(\omega)$ of the generalized coordinate q_j as:

$$S_{q_j}(\omega) = \frac{S_{F_j}(\omega) |X_j|^2}{\omega^4 m^{*2}} \quad (7)$$

where $S_{F_j}(\omega)$ is the power spectrum of modal force, F_j ; ω is the circular frequency; and

$|X_j|^2 = \left[1 / \left\{ (1 - (\omega/\omega_j)^2)^2 + 4\zeta_j^2 (\omega/\omega_j)^2 \right\} \right]$ represents the mechanical admittance function of mode j of the building with a natural frequency $= \omega_j$.

According to spectral analysis theory, the area under the displacement spectrum is equal to the square of the root-mean-square (rms) quantity (i.e. the variance) of the measurement. Figure B1 illustrates superimposed spectrum of the generalized force (dark blue) and spectrum of the generalized displacement (light blue). Hence, the variance of displacement is obtained as follows based on Square Root of Square Sum (SRSS) method:

$$\sigma_x^2 = \sum_j \left[\phi_{xj}^2 \int_0^\omega S_{qj} d\omega \right] \quad (8)$$

In a similar way since acceleration and displacement are related by ω^2 acceleration response is given by

$$\sigma_{ax}^2 = \sum_j \left[\phi_{xj}^2 \int_0^\infty \omega^4 S_{qj} d\omega \right] \quad (9)$$

Peak displacement and accelerations for each principal axis are obtained from the corresponding root-mean-square values (σ_x or σ_{ax}) by multiplying by the appropriate peak factor which is generally in the range of 3 to 4.

The dynamic components of base moments and forces are evaluated from the following summations over tributary area i , and mode j :

$$M_x \cong -\sum_i F_{yi} z_i + \sum_j \int_0^H \left[\phi_{xj}(z) m(z) \frac{d^2 q_j}{dt^2} z \right] dz; \quad (10)$$

$$M_y \cong \sum_i F_{xi} z_i - \sum_j \int_0^H \left[\phi_{yj}(z) m(z) \frac{d^2 q_j}{dt^2} z \right] dz; \quad (11)$$

$$T \cong \sum_i F_{\theta i} - \sum_j \int_0^H \left[\phi_{\theta j}(z) I(z) \frac{d^2 q_j}{dt^2} z \right] dz; \quad (12)$$

$$S_x \cong \sum_i F_{xi} - \sum_j \int_0^H \left[\phi_{xj}(z) m(z) \frac{d^2 q_j}{dt^2} \right] dz; \quad (13)$$

$$S_y \cong \sum_i F_{yi} - \sum_j \int_0^H \left[\phi_{yj}(z) m(z) \frac{d^2 q_j}{dt^2} \right] dz. \quad (14)$$

It is difficult to extract torsional modal force which has non-constant mode shape based on HFFB method ² which inherently measures base torque with a constant mode shape (influence function). However, HFPI can precisely evaluate the torsional modal force by considering the wind load variation along the height from the pressure integration as described in equations (2) and (5).

The mean and root-mean-square values of moments obtained from the model study are scaled to full-scale by

$$M_p = M_m \lambda_L^3 \lambda_U^2, \text{ while}$$

the mean and root-mean-square values of forces are scaled by

$$F_p = F_m \lambda_L^2 \lambda_U^2$$

Where subscripts m and p refer to model and full-scale values respectively, λ_L refers to length scale (L_m/L_p) and λ_U is the velocity scale (U_m/U_p) of the model.

References

- 1) Reinhold, T.A., Measurement of Simultaneous Fluctuating Loads of Multiple Levels of a Model of a Tall Building in a Simulated Urban Boundary Layer, PhD Thesis, Virginia Polytechnic Institute and State University, November 1977.
- 2) Tschanz, T., The Base Balance Measurement Technique and Applications to Dynamic Wind Loading of Structures, PhD Thesis, The University of Western Ontario, Faculty of Engineering Science, London, Ontario, September 1982.

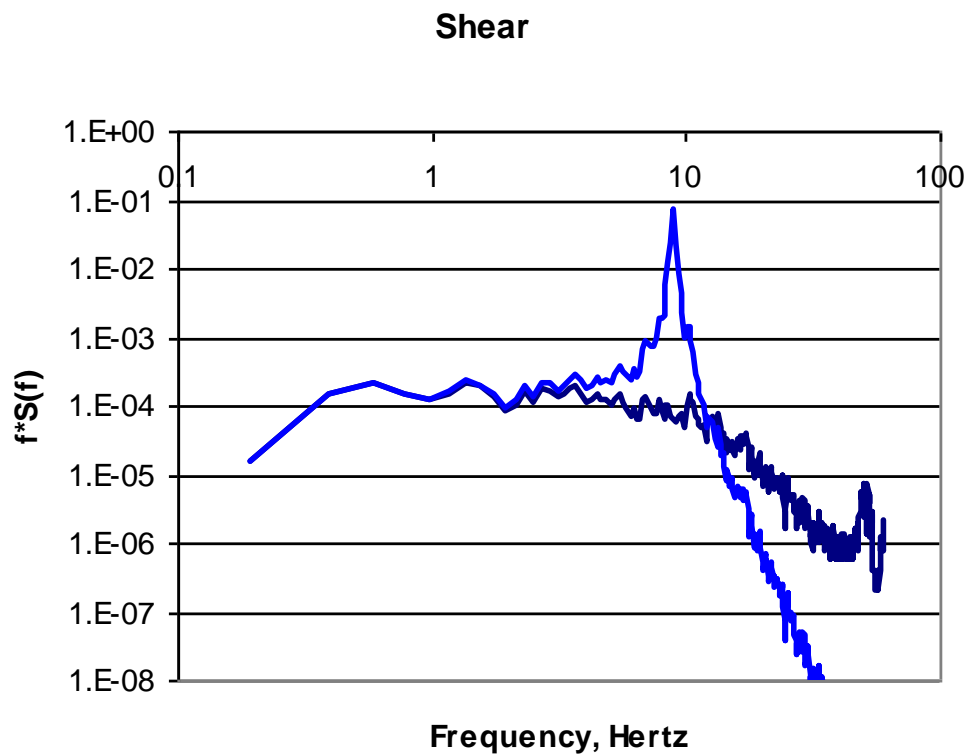


Figure B1: Spectra of Force (Dark) and Displacement (Light)

APPENDIX C

WIND LOAD VARIATION WITH HEIGHT

WIND LOAD VARIATION WITH HEIGHT

Wind loads on flexible structures produce mean and dynamic responses. The mean loading can be obtained in one of several ways including a cladding pressure test. The dynamic component can be considered to arise from two distinct effects including, a quasi-steady loading and a fluctuating effect. Whereas the quasi-steady loading arises from the correlation of wind forces over the surface of the building, the fluctuating forces arise as the building mass responds to the applied forces at its natural frequency and mode of vibration. This latter quantity is the resonant component of the response. In spite of the different origins, the dynamic loads can be interpreted in the form of equivalent static load variation as a function of height arising from the inertial forces of motion in sway and torsion.

Hence, if we consider the force per unit height in either sway direction, $f_x = \mu(z)a_x$, where $\mu(z)$ is the mass per unit height z and a_x is the sway acceleration, then the peak force is

$$\hat{f}_x = K \cdot \mu(z) \cdot \phi(z) \cdot \omega^2 \quad (1)$$

The new terms in equation (1) are, K = constant to be determined, $\phi(z)$ is the mode shape, and ω^2 is the square of the natural frequency corresponding to $\phi(z)$. K and ω^2 being both constants can be combined into a new constant K_2 , which is determined from the fact that the forces in equation (1) must be compatible with the base bending moment, M_B . Hence,

$$M_B = \int_0^H \hat{f}_x(z) \cdot z \cdot dz \quad (2)$$

Substituting equation (1) into equation (2) gives

$$K_2 = \frac{M_B}{\int_0^H \mu(z) \cdot \phi(z) \cdot z \cdot dz} \quad (3)$$

If the mode shape is linear and expressed as $\phi(z) = z/H$, equation (3) can be written

$$K_2 = \frac{M_B}{H \cdot \int_0^H \mu(z) \cdot \phi^2(z) \cdot dz} \quad (4)$$

And since the integral is equal to the generalized mass, m^* , K_2 becomes,

$$K_2 = M_B / (H \cdot m^*) \quad (5)$$

Substituting equation (5) into equation (1) with $K_2 = K \cdot \omega^2$ gives

$$\hat{f}_x = M_B \cdot \mu(z) \cdot z / (H^2 \cdot m^*) \text{ or for discrete floors} \quad (6)$$

$$\hat{f}_i = M_B \cdot m_i \cdot z_i / (H^2 \cdot m^*) \quad (7)$$

The variation of torsional loads can be obtained with similar reasoning. The inertia force created by twisting motion of the building creates a torque, $t_z(z)$ per unit height expressed as:

$$t_z(z) = MMI(z) \cdot a_\theta \quad (8)$$

where $MMI(z)$ is the polar mass moment of inertia as a function of height, and a_θ is the rotational acceleration of the building in its torsional mode. The torsional moment on a floor by floor basis similar to equation (7) is given by equation (9) as follows:

$$\hat{t}_z(z_i) = (T_B \cdot MMI(z_i) \cdot z_i / H_i) \cdot \int_0^H MMI(z) \cdot \phi(z) \cdot dz$$

where subscript i refers to quantities at floor level i . Alternately, for compact floor plans, (i.e. floor plans that are close to be doubly symmetric), a suitable alternate approach is to use the sway forces offset from the centre of twist by an amount sufficient to produce the measured base torque.