SPAN BALANCE ANALYSIS REPORT

Test Conducted: May 10, 2017

BURLINGTON CANAL VERTICAL LIFT BRIDGE
HAMILTON, ONTARIO
CANADA
PWGSC Project No. R.012641.001

Submitted to:

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Submitted: May 26, 2017

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INTRODUCTION

This report details balance tests performed at the Burlington Canal Vertical Lift Bridge in Hamilton, Ontario, Canada. The tests were conducted by Stafford Bandlow Engineering, Inc. (SBE) for The State Group, Inc. to obtain the post construction balance condition of the lift span at the end of a rehabilitation project.

SBE was on site at the Burlington Canal Vertical Lift Bridge on May 10, 2017 for the purpose of performing balance tests. The tests were conducted on the span drive machinery in the North and South towers simultaneously during testing. Figure 1, Appendix A depicts a general plan of the lift span with identification of directions.

TEST PROCEDURE AND EQUIPMENT

The current tests were performed utilizing gages that were installed by SBE as part of prior balance tests at the bridge. The gages were installed as follows: Two 2-element (gage) 90 degree foil type strain gage rosettes were spot welded to each of the eight main pinion shafts that engage the sheave's ring gear. Figure 2, Appendix A depicts the layout of the span drive machinery and shows the location of the strain gage installation. A total of eight rosettes (sixteen gages) were used for each tower. Measurements Group LWK-06-W250D-350 gages were used. Surface preparation of the shaft and mounting of the gages were performed in accordance with the strain gage manufacturer's requirements. Each pair of rosettes was mounted on a circumferential line 180 degrees apart from one another. The gages from each pair of rosettes were wired in a full Wheatstone bridge; the Wheatstone bridge effectively cancels out strain on the shaft surface produced by bending and temperature changes and ensures that the indicated strain is due to torsion only. Once mounted the gages were protected for future use. The gages were checked for proper resistance and grounding prior to the current test. All gage values were found to be within an acceptable range.

Each Wheatstone bridge was hard wired via Belden twisted pair shielded cable to one channel of a four channel Somat e-DAQ-lite bridge expansion board. Figure 3, Appendix A depicts the strain gage wiring schematic. The gain for each channel was set such that the relation between the output from the system in millivolts and the shear strain at the surface of the shaft in microstrain was established. nCode's Test Control Environment software was used to capture the requisite data.

Prior to recording actual test data, the brakes were released to relieve any torsional strain in the shafts due to residual or seating torque from the span drive machinery. The motor shaft was the rotated to obtain clearance on each side of the engaged main pinion teeth. Clearance was verified with a feeler gage. Once clearance was obtained, the instruments were zeroed with the shaft torque set to zero.

Lift height was monitored through event marks indicating revolutions of the main pinion shaft. The event marks were provided by a Hall Effect sensor which monitored two diametrically opposite magnets affixed to the pinion shaft and provided a voltage output each time a magnet passed the sensor. Calculations were performed to convert revolutions of the main pinion shaft into the change in lift height in feet. Two magnets were affixed to the pinion shaft so that each event mark corresponded to 1.6 feet of lift height, and 68 events were provided for a normal full lift (109 ft.). The Hall Effect sensor readings were taken at the same tower where strain readings were recorded.

Each strain channel was sampled at a rate of 50 Hz for the duration of each bridge lift. The data was reviewed in the field at the conclusion of each bridge operation to check the integrity of the data and then saved to disk.

METHOD OF ANALYZING RECORDED DATA

Strip charts for each run from each tower of the testing are presented in Appendix B. Each strip chart contains the strain data for the four instrumented shafts in each tower versus the lift height for a complete opening cycle. The data from each test run was analyzed at incremental lift heights which corresponded to the system event marks. 100 data points centered on each event mark are summed and then averaged. In this way, any periodic fluctuations in the test data (sliding friction on the gear teeth, gear tooth impacts, etc.) are effectively filtered out. Averaged data points were then selected from the constant velocity region of each test run; the data in the accelerating and decelerating regions were discarded. The averaged data points and their corresponding lift heights were entered into a proprietary balance program and used to determine the balance condition of the lift span according to the governing balance equation. These calculations are described below.

The strain recorded in the shaft relates to the torque in the shaft according to the mechanics of the shaft geometry and material. This allows the shaft torque to be calculated from the recorded strain data. Then the ratio between the main pinion and the sheave trunnion is used to convert main pinion shaft torque to sheave trunnion torque. An efficiency factor is used to account for the frictional losses between the rack pinion shaft and the sheave trunnion. This factor is calculated using the machinery losses and efficiency coefficients provided in the Canadian Highway Bridge Design Code, CAN/CSA-S6-06 (hereafter referred to as CHBDC) Table 13.8. The sheave trunnion torque was then converted to an equivalent force at the main counterweight ropes using the pitch radius of the ropes.

The imbalance force was determined in the following manner: For a given span position the imbalance assists the machinery in one direction (raise or lower) and resists the machinery in the opposite direction. Friction always opposes the machinery. Therefore, the summation of the raising and lowering force at a given lift height divided by two is equal to the span imbalance force at that lift height. This assumes that the friction is equal in both directions. Since there is no reasonable way to determine the true system friction this assumption must be made.

The span balance changes with lift height as a result of the main counterweight ropes passing over the counterweight sheaves and due to the effect of the auxiliary counterweight system. A mathematical equation for the theoretical change in span balance versus lift height due to both factors was derived based on the geometry and weights provided on the original design drawings, with two noteworthy exceptions.

1. The weight of each auxiliary counterweight has been modified from 18,000 lbs. as indicated on the design drawings to 18,500 lbs. in accordance with changes during a 2002-2003 main counterweight rope replacement project.
2. The weight of the main counterweight ropes has been taken as 8.51 lbs. per linear foot in accordance with standard rope manufacturer's information.

The mathematical equation is used in a curve-fitting program to determine the best fit of the theoretical imbalance curve to the imbalance data. The fitted imbalance curve is then used to determine the imbalance at the fully lowered position. Actual imbalance data cannot be obtained in the lowered position due to the acceleration torque. Sample calculations utilizing the balance equation are presented in Appendix C. Figure 4, Appendix A, identifies the auxiliary counterweight system variables used in the balance equation.

WEATHER CONDITIONS

The weather on May 10, 2017 was clear and dry with a mean temperature of 46°F. The average wind speed reported at the Hamilton, ON weather station was 3 MPH with gusts up to 6 MPH at the time of the testing.

PRESENTATION OF BALANCE RESULTS

The following table documents the balance condition for each corner of the lift span. The table presents the seated imbalance (i.e. imbalance with span fully seated) as well as the friction force, which is also determined as part of the analysis. The individual results from each of three test runs are provided with the averages of the three test runs.

Burlington Canal Lift Bridge
North Tower
Test Date: May 10, 2017

	Seated Imbalance (lb.)			Average Friction (lb.)		
Run	**NW Corner**	**NE Corner**	**North End**	**NW Corner**	**NE Corner**	**North End**
1	+4,539	+2,868	+7,407	+2,857	+2,873	+5,731
2	+6,285	+1,226	+7,512	+3,319	+2,485	+5,804
3	+6,559	+871	+7,430	+3,442	+2,314	+5,756
Average	**+5,794**	**+1,655**	**+7,450**	**+3,206**	**+2,557**	**+5,764**

Burlington Canal Lift Bridge
South Tower
Test Date: May 10, 2017

	Seated Imbalance (lb.)			Average Friction (lb.)		
Run	**SW Corner**	**SE Corner**	**South End**	**SW Corner**	**SE Corner**	**South End**
1	+5,050	+1,241	+6,291	+3,768	+1,908	+5,676
2	+2,428	+3,971	+6,399	+3,119	+2,677	+5,796
3	+225	+5,974	+6,199	+2,628	+3,064	+5,692
Average	**+2,568**	**+3,729**	**+6,296**	**+3,172**	**+2,550**	**+5,721**

Note: Positive (+) imbalance indicates span heavy.
Negative (-) imbalance indicates counterweight heavy.

The fitted imbalance curves which yield the above results are presented in Appendix D. Each graph contains the best fit of the theoretical imbalance curve to the imbalance data, the opening force, the closing force and the friction force relative to the main counterweight ropes.

DISCUSSION OF RESULTS

The Specifications establish requirements for final balance as follows:

> *"Make any necessary adjustment to the counterweights required to achieve a balance condition equal to that measured prior to construction (plus or minus 5%)."*

The following table summarizes the average seated imbalance per tower at the time of the initial test on April 22, 2015 and at the time of the post construction test on May 10, 2017:

Burlington Canal Lift Bridge Results Comparison		
Test Date	**Average Seated Imbalance (lb.)**	
	North Tower	**South Tower**
April 22, 2015	+7,452	+6,265
May 10, 2017	+7,450	+6,296
% Difference	-0.03%	0.49%

The post construction balance condition of each end of the lift span is within 0.5% of the balance condition prior to construction, which is well within the acceptance criteria in the Specifications.

APPENDIX A

Figures

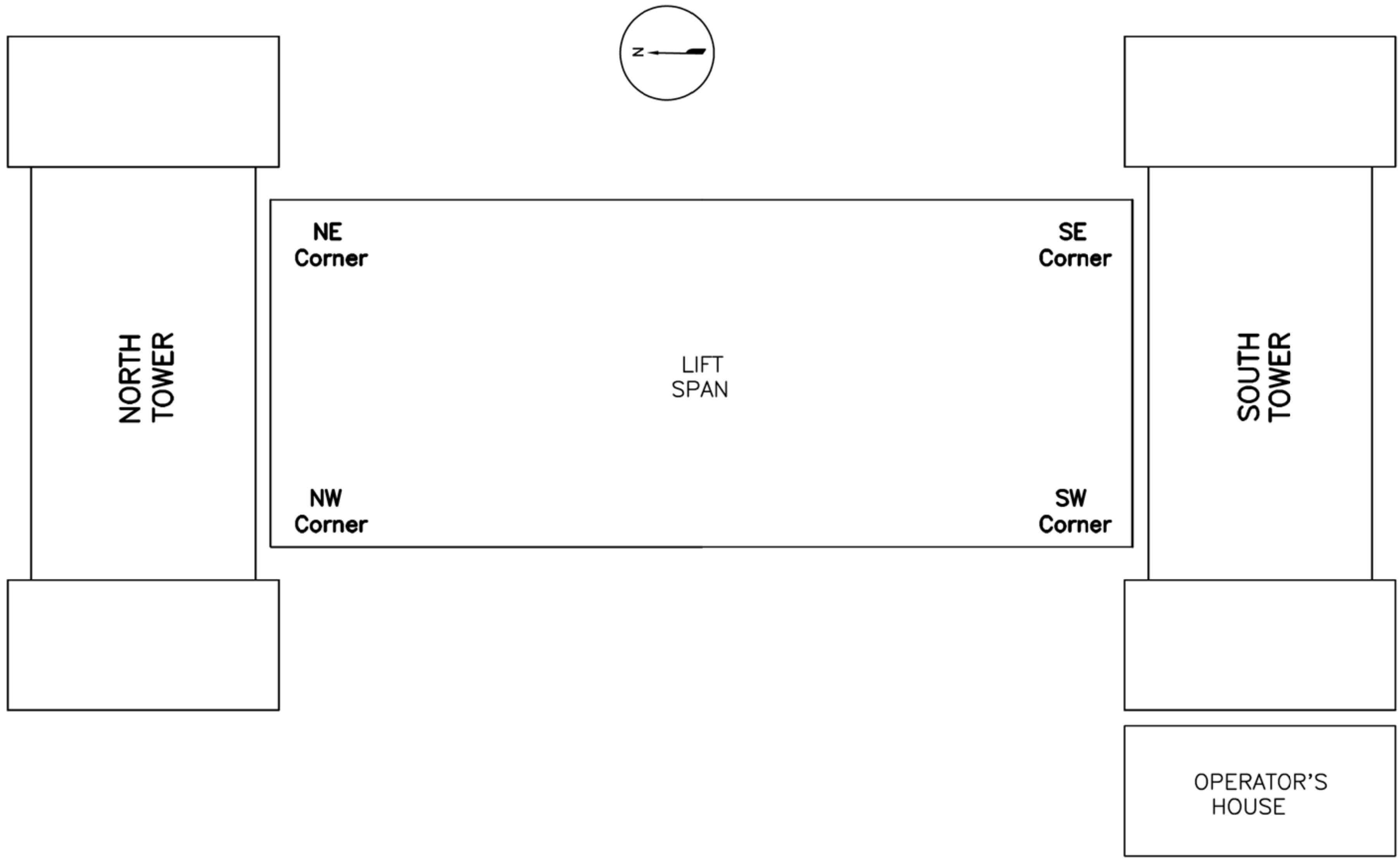
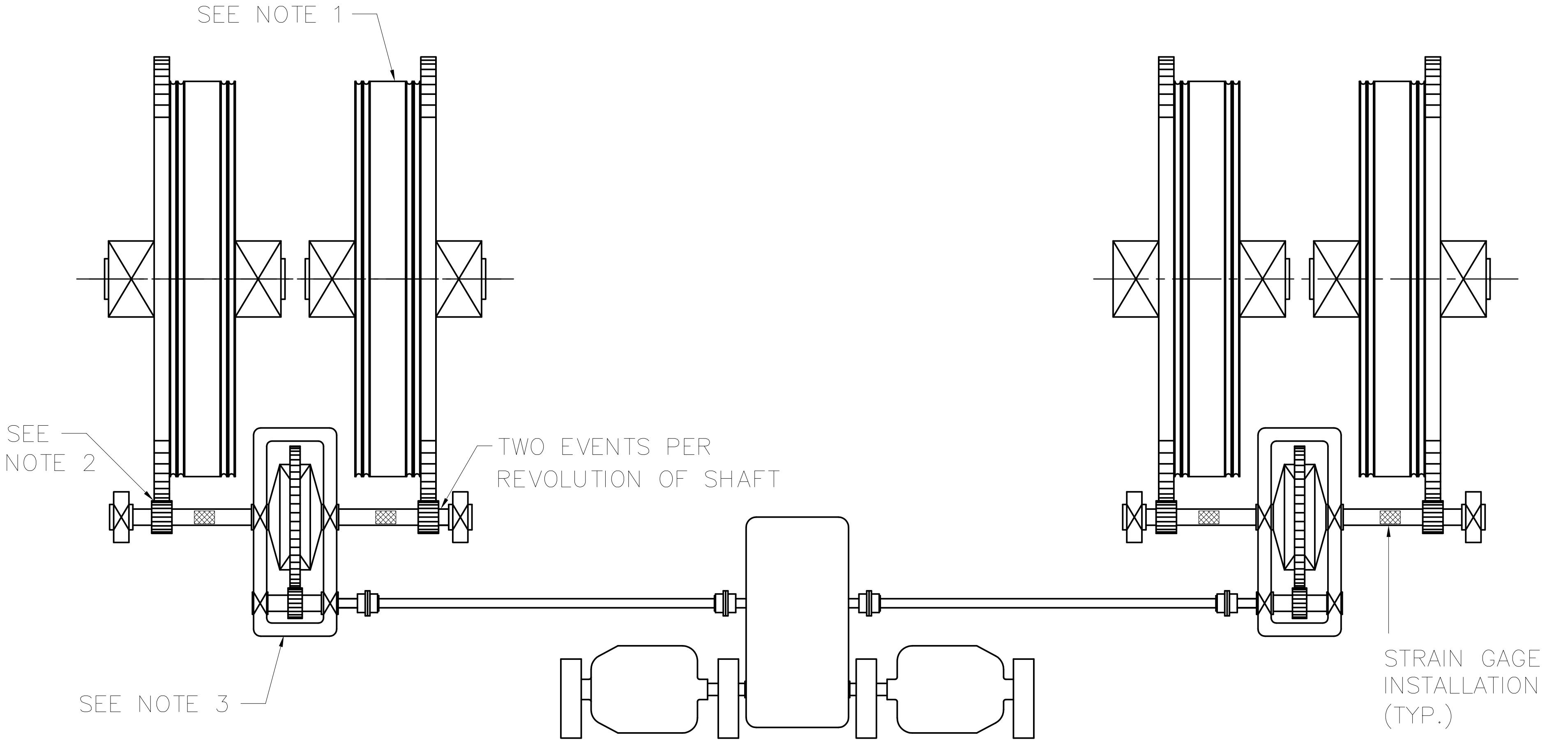


Figure 1. Plan View of Movable Span

Project:
Burlington Canal Lift Bridge
Hamilton, Ontario
Strain Gage Imbalance Tests

Prepared by:
Stafford Bandlow Engineering, Inc.



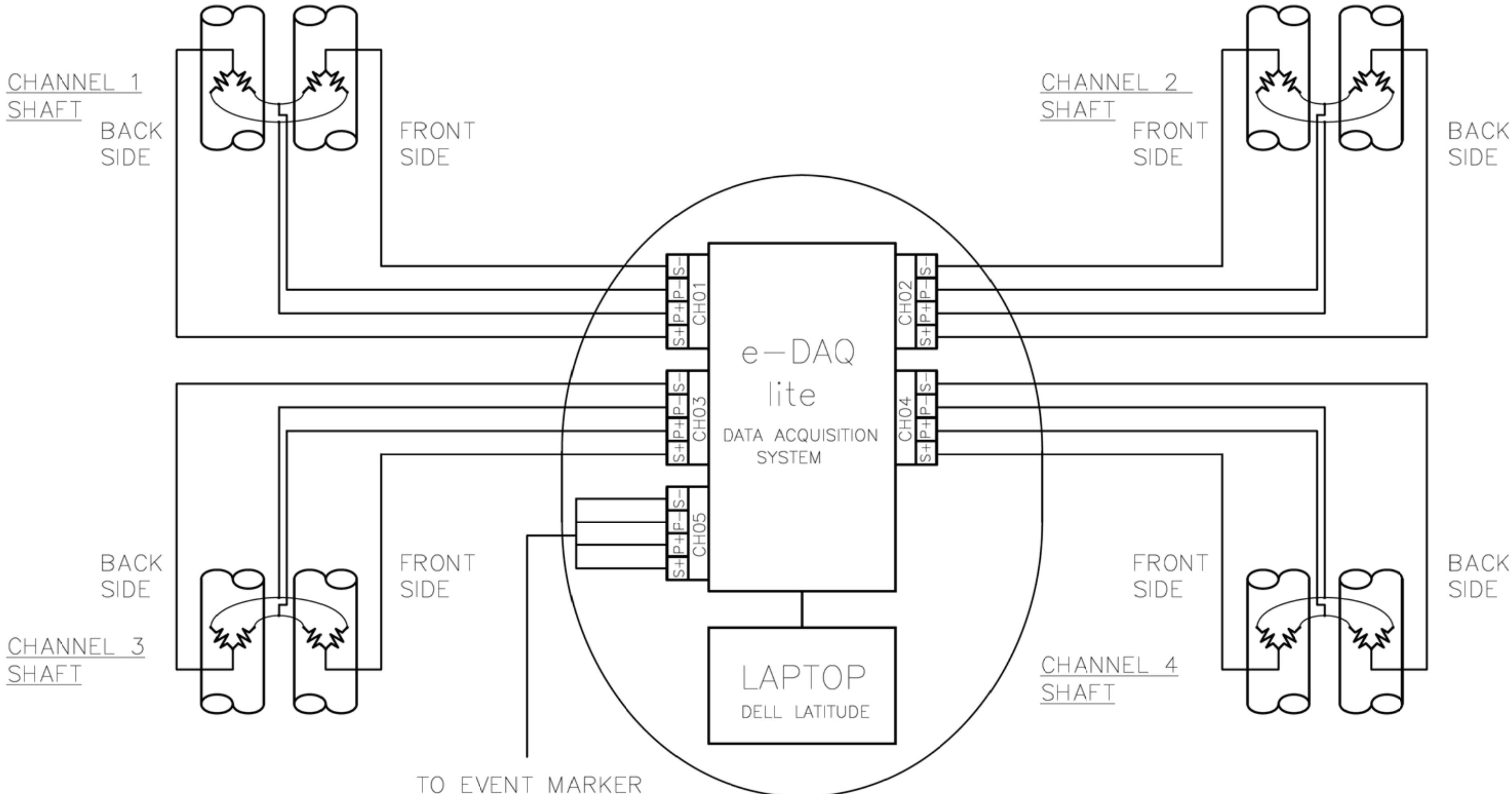
NOTES:
1. ROPE PITCH DIA. ON SHEAVE = 15'
2. RATIO RACK PINION/RING GEAR = 14.74
3. RATIO SECONDARY REDUCER= 4.21

NORTH TOWER SHOWN
(SOUTH TOWER SIMILAR)

Figure 2. Location of Strain Gage Installation and Event Mark Recording

Project:
Burlington Canal Lift Bridge
Hamilton, Ontario
Strain Gage Imbalance Tests

Prepared by:
Stafford Bandlow Engineering, Inc.



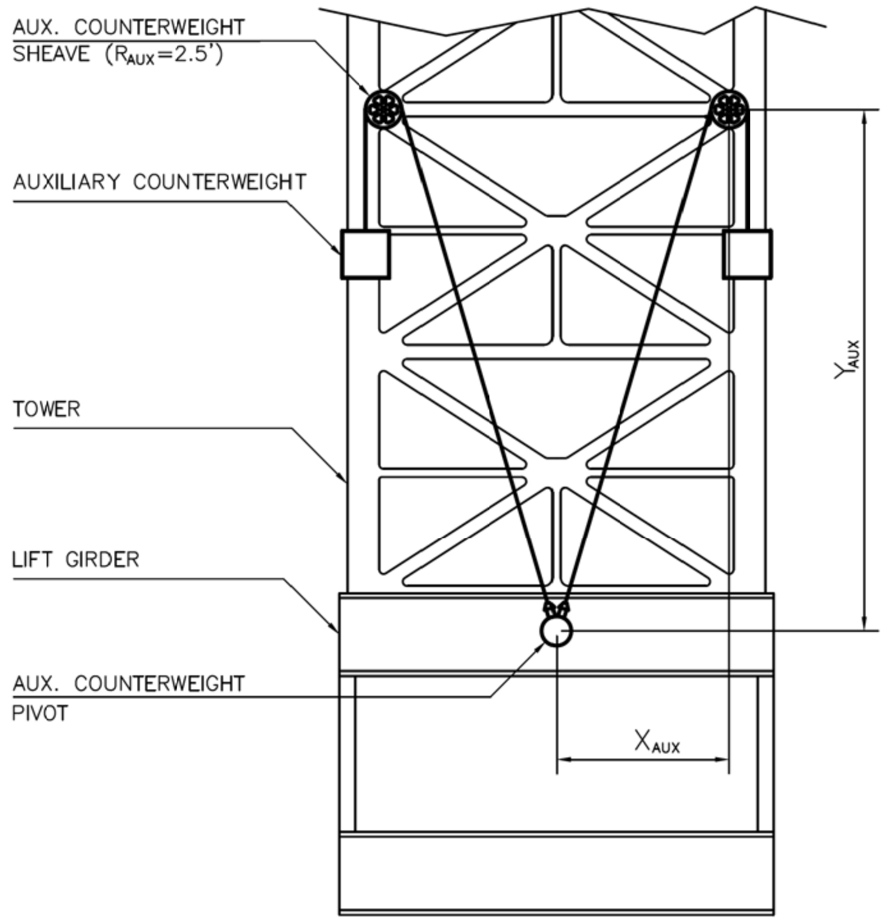
NOTE:
1. THE SETUP SHOWN IS FOR ONE TOWER ONLY.

KEY
CH: CHANNEL
P: POWER
S: SIGNAL

Figure 3. Strain Gage Wiring Schematic

Project:
Burlington Canal Lift Bridge
Hamilton, Ontario
Strain Gage Imbalance Tests

Prepared by:
Stafford Bandlow Engineering, Inc.



X_{AUX} = 23.167 ft.
Y_{AUX} = 48.286 ft.
R_{AUX} = 2.5 ft.

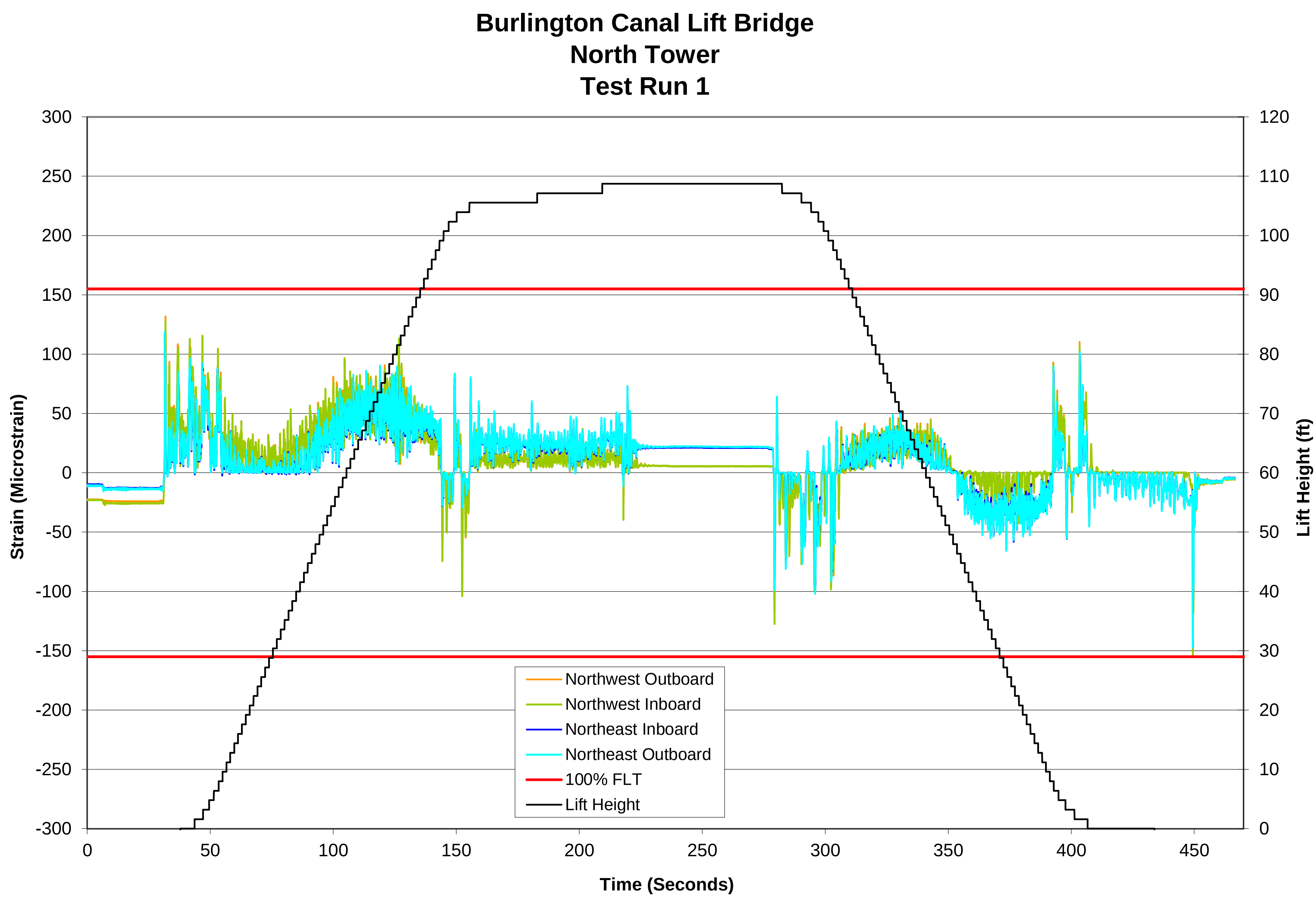
Figure 4. Auxiliary Counterweight Machinery

Project:
Burlington Canal Lift Bridge
Hamilton, Ontario, Canada
Strain Gage Imbalance Tests

Prepared by:
Stafford Bandlow Engineering, Inc.

APPENDIX B

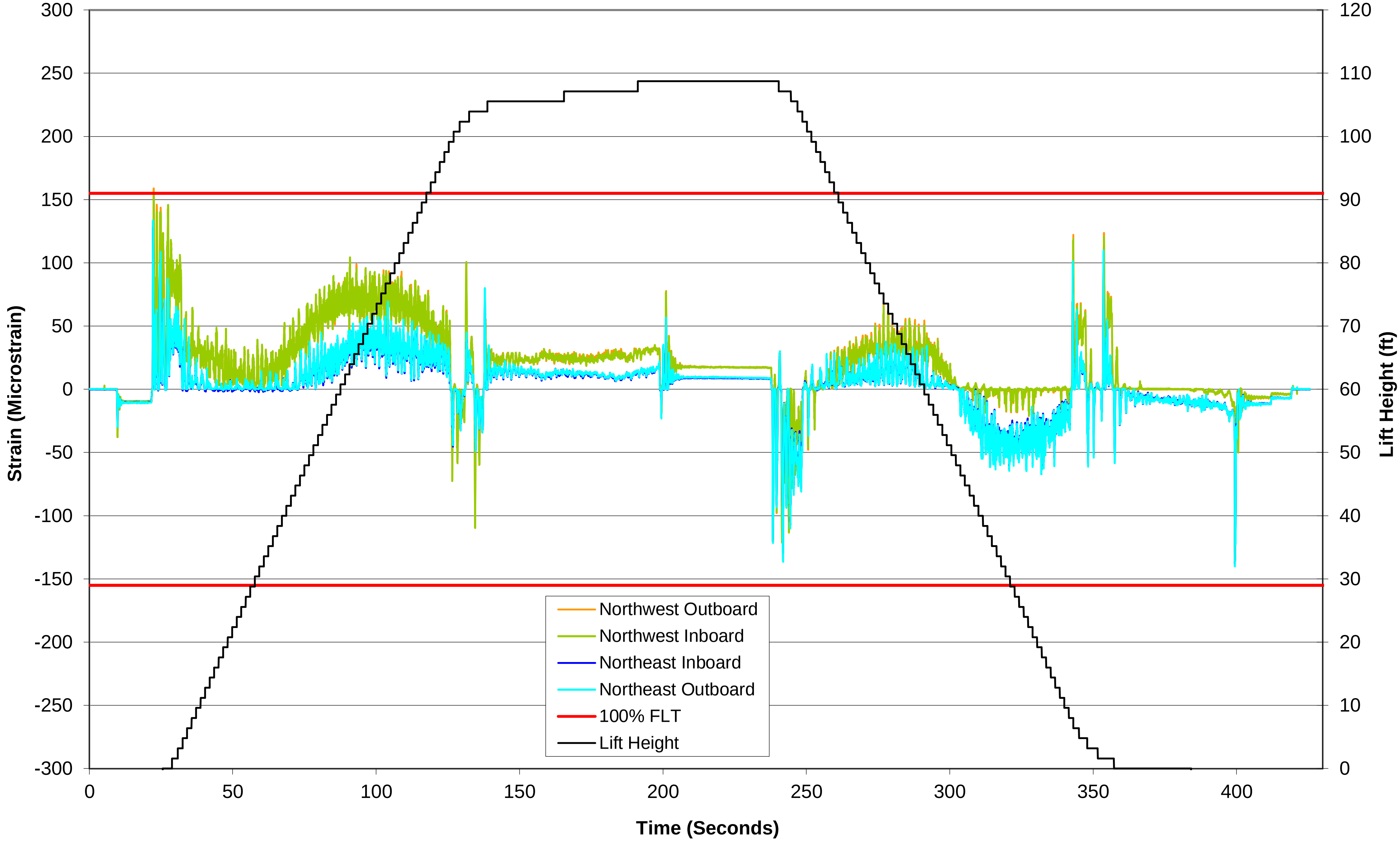
Strip Chart Recordings



Prepared by: Stafford Bandlow Engineering, Inc.
Prepared for: The State Group

Date Recorded: May 10, 2017

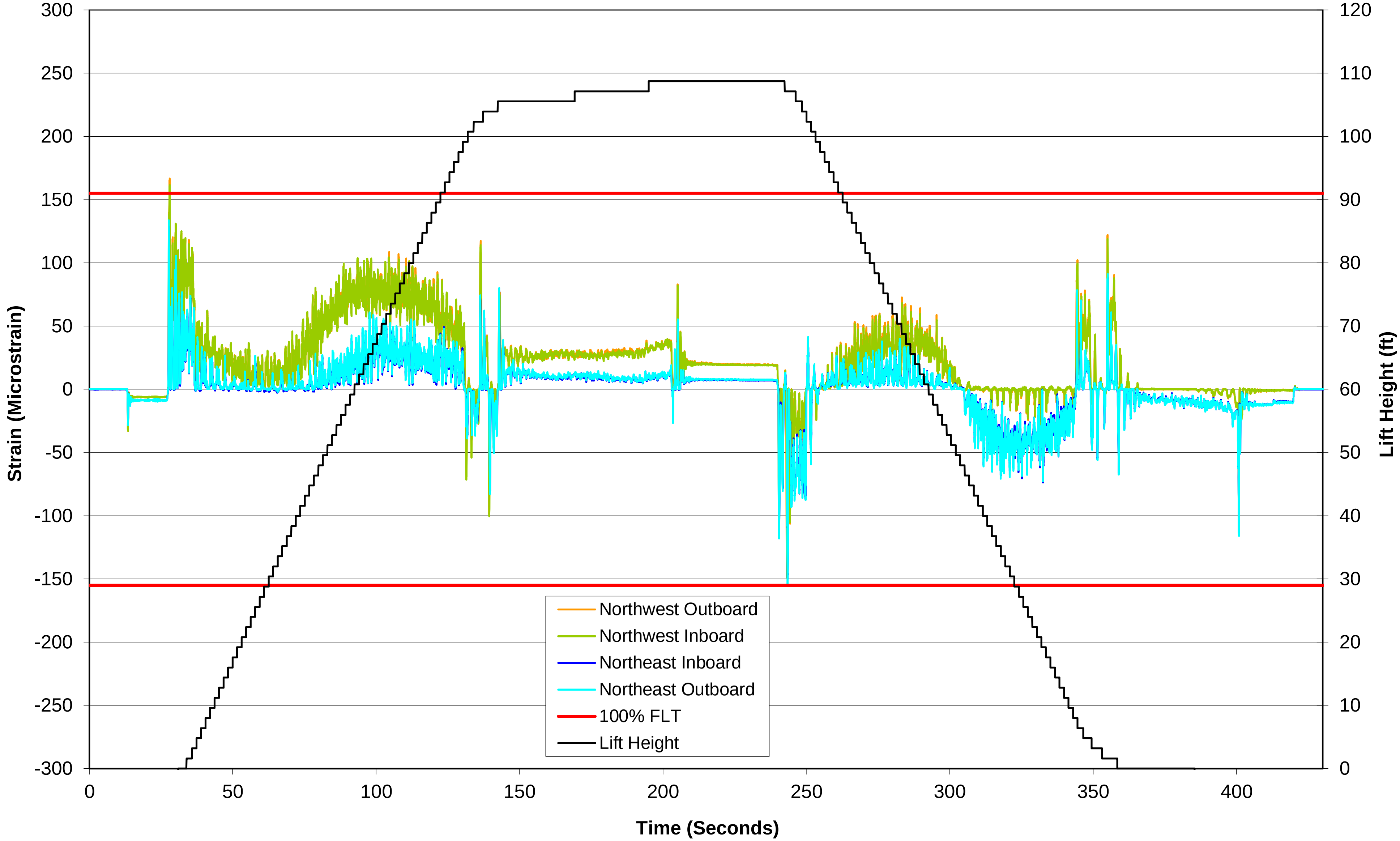
Burlington Canal Lift Bridge
North Tower
Test Run 2



Prepared by: Stafford Bandlow Engineering, Inc.
Prepared for: The State Group

Date Recorded: May 10, 2017

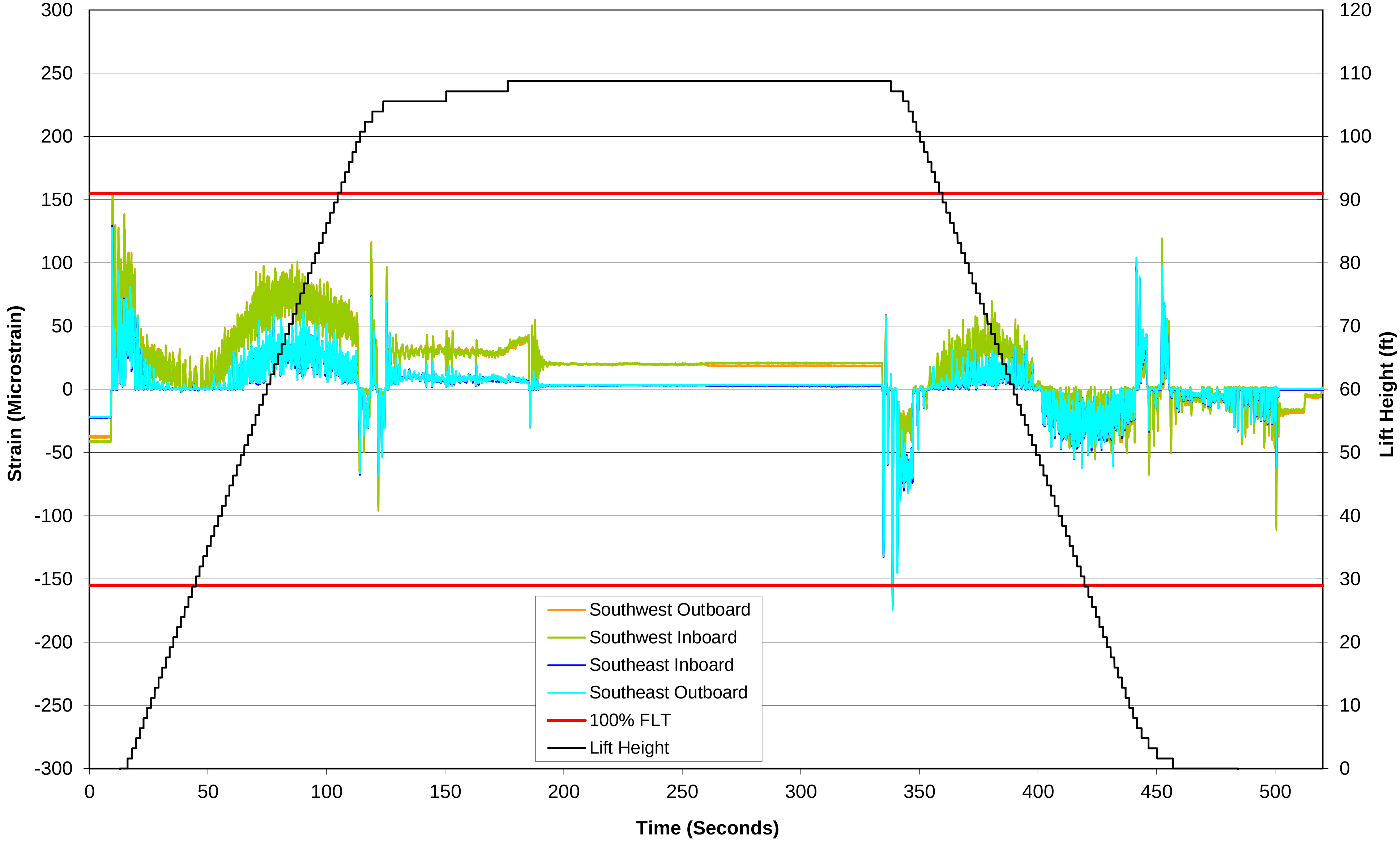
Burlington Canal Lift Bridge
North Tower
Test Run 3



Prepared by: Stafford Bandlow Engineering, Inc.
Prepared for: The State Group

Date Recorded: May 10, 2017

Burlington Canal Lift Bridge
South Tower
Test Run 1

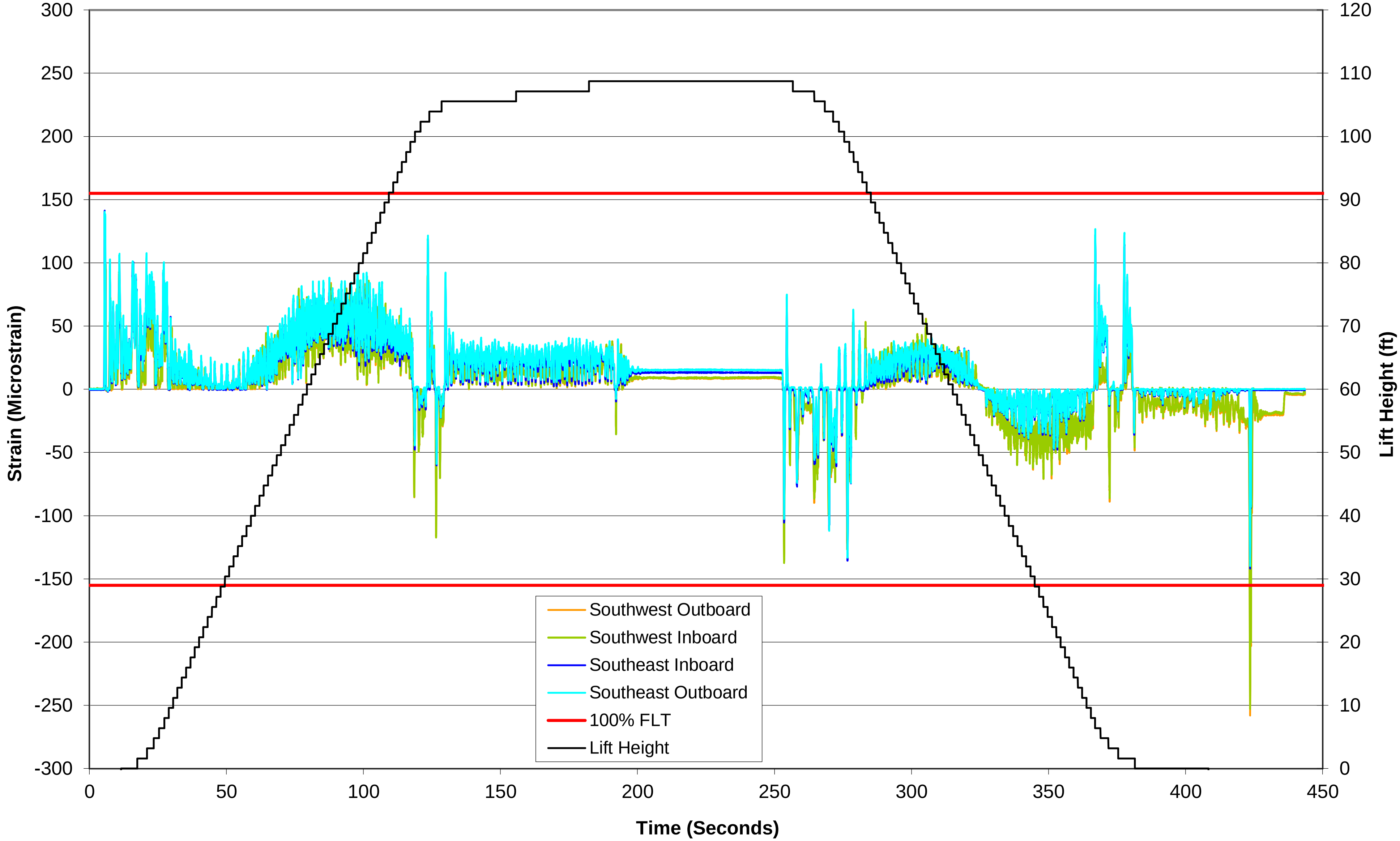


Prepared by: Stafford Bandlow Engineering, Inc.

Prepared for: The State Group

Date Recorded: May 10, 2017

Burlington Canal Lift Bridge
South Tower
Test Run 2



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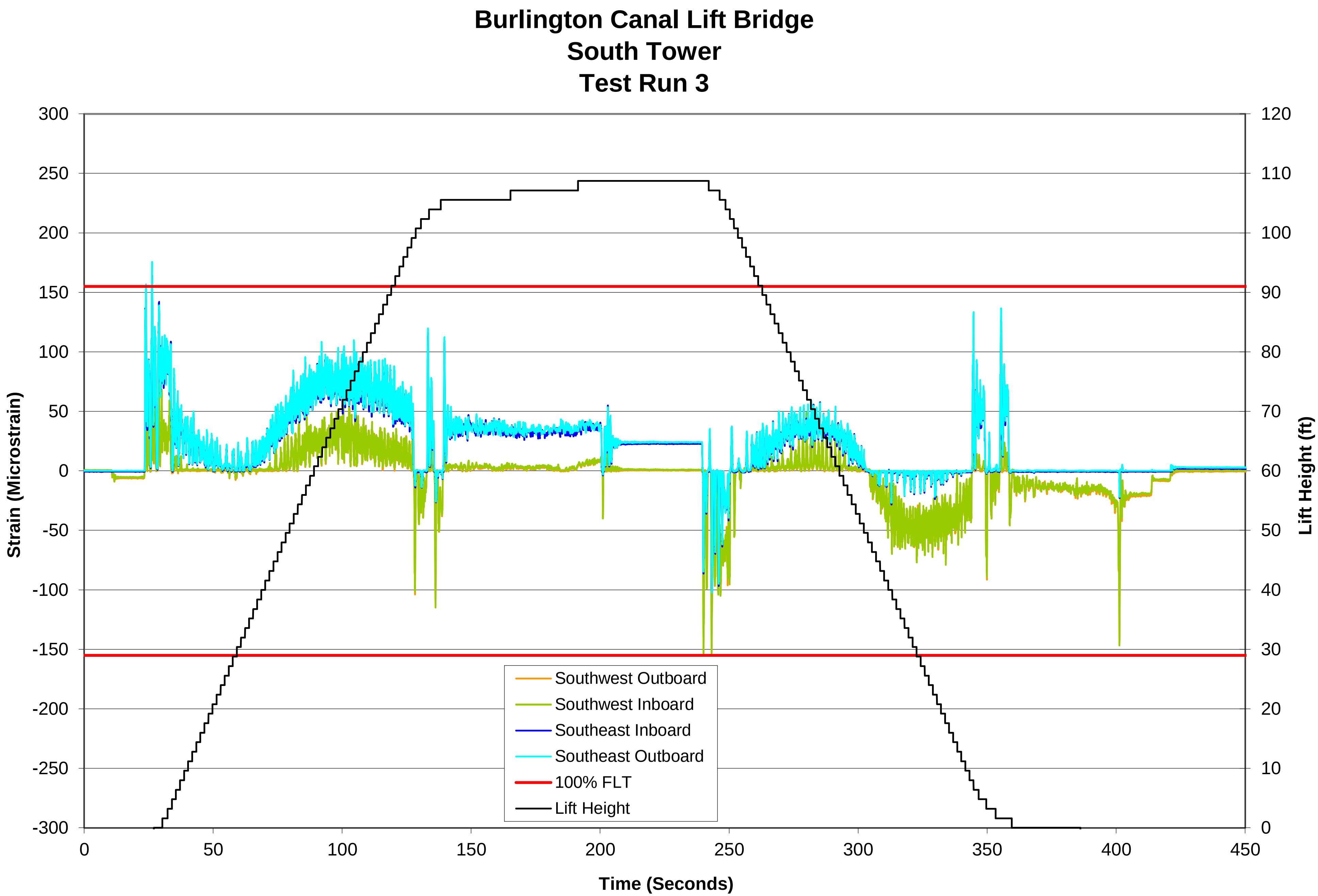


Prepared by: Stafford Bandlow Engineering, Inc.
Prepared for: The State Group

Date Recorded: May 10, 2017

APPENDIX C

Sample Balance Calculations

Governing Equation
(for each corner)

$$IMB_i = \left[Imbalance_{set} + \left[-2 \cdot (corner \cdot Ropes \cdot Mass_{unit}) \cdot height_i \right] \ldots + corner \cdot \left[(-1) \cdot F_{cwt} \cdot \left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_i}{x_{aux}} \right) + \operatorname{asin}\left[\frac{R_{aux}}{\left[\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_i}{x_{aux}} \right) \right)} \right]} \right] \right] \right] \right] \right]$$

Known:

$F_{cwt} := 18500$ $\quad Ropes := 20$ $\quad corner = 2$ $\quad x_{aux} := 23.167$

$Mass_{unit} := 8.51$ $\quad R_{aux} := 2.5$ $\quad y_{aux} := 48.286$

Reference Figure 4 for identification of X_{AUX}, Y_{AUX}, R_{AUX}.

Input Data from strip charts:
Note: For presentation, a limited portion of the data used in the calculation is shown below. A total of 69 data points are used in the analysis.

	event =	$CH1_{op}$ =	$CH2_{op}$ =	$CH3_{op}$ =	$CH4_{op}$ =
	0	0	0	0	0
0	0	35.5	34.97	9.36	1.48
1	1	40.99	39.07	12.5	5.31
2	2	102.51	100.99	74.63	65.37
3	3	31.75	30.35	12.21	4.7
4	4	29.8	29.31	12.2	4.7
5	5	31	29.47	10.39	3.23
6	6	27.59	27.18	11.21	3.78
7	7	27.42	26.4	9.05	1.63
8	8	24.95	24.54	7.72	-0.36
9	9	21.73	20.56	8.53	0.97
10	10	19.68	19.28	7.73	-0.23
11	11	16.94	16.08	7.49	-0.18
12	12	13.84	13.37	7.26	-0.76
13	13	12.37	11.46	7.17	-0.44
14	14	9.8	9.25	7.43	-0.79
15	15	7.94	6.97	7.14	-0.09

$CH1_{cl} =$

	0
0	6.85
1	36.67
2	5.37
3	3.49
4	1.8
5	1.48
6	49.43
7	-1.13
8	-1.43
9	-1.5
10	-2.03
11	-2.1
12	-3.68
13	-2.88
14	-5.42
15	-4.32

$CH2_{cl} =$

	0
0	6.21
1	35.04
2	4.78
3	2.54
4	1.18
5	0.58
6	49.27
7	-1.74
8	-1.91
9	-2.02
10	-2.53
11	-2.6
12	-4.26
13	-3.31
14	-6.01
15	-5.01

$CH3_{cl} =$

	0
0	-6.42
1	15.91
2	4.61
3	2.56
4	0.65
5	-3.72
6	39.4
7	-7.15
8	-9.52
9	-13.08
10	-15.9
11	-18.31
12	-19.56
13	-22.63
14	-21.07
15	-25.87

$CH4_{cl} =$

	0
0	-11.91
1	10.24
2	-1.84
3	-2.68
4	-5
5	-9.49
6	32.38
7	-12.34
8	-15.35
9	-18.83
10	-22.03
11	-24.12
12	-25.86
13	-28.52
14	-27.73
15	-31.18

$$\text{height} := \text{event} \cdot \left[\frac{\pi \cdot 15}{\left(\frac{280}{19}\right)} \cdot \frac{1}{2} \right]$$

This operation converts the event marks to lift height in feet (2 event marks per revolution of the rack pinion shaft).

The following variables will be used:

G = shear modulus of shaft material (11,500,000 psi for steel)
J = polar moment of inertia
R = total ratio from gages to trunnion including rack and pinion
Ro = radius to outside of shaft on which gages are mounted
Ri = radius of hole through shaft (if applicable)
Ratio = Ratio from pinion to rack
Radius to ropes = Rope pitch radius on sheave
n = efficiency from gages to trunnion

$$G := 11.5$$

$$R_o := \frac{6.5}{2}$$

$$R_i := \frac{0}{2}$$

$$J := \frac{\pi}{2} \cdot \left(R_o^{\,4} - R_i^{\,4}\right)$$

Solve for Torque in shaft on which gages are located:
TOS = opening torque in the shaft on which the gages are mounted
TCS = closing torque in the shaft on which the gages are mounted

$$TOS_{ch1} := G \cdot \frac{J}{R_o} \cdot CH1_{op} \cdot \frac{1}{12} \qquad TCS_{ch1} := G \cdot \frac{J}{R_o} \cdot CH1_{cl} \cdot \frac{1}{12}$$

$$TOS_{ch2} := G \cdot \frac{J}{R_o} \cdot CH2_{op} \cdot \frac{1}{12} \qquad TCS_{ch2} := G \cdot \frac{J}{R_o} \cdot CH2_{cl} \cdot \frac{1}{12}$$

$$TOS_{ch3} := G \cdot \frac{J}{R_o} \cdot CH3_{op} \cdot \frac{1}{12} \qquad TCS_{ch3} := G \cdot \frac{J}{R_o} \cdot CH3_{cl} \cdot \frac{1}{12}$$

$$TOS_{ch4} := G \cdot \frac{J}{R_o} \cdot CH4_{op} \cdot \frac{1}{12} \qquad TCS_{ch4} := G \cdot \frac{J}{R_o} \cdot CH4_{cl} \cdot \frac{1}{12}$$

Solve for Force at Ropes to produce recorded torques
FOS = opening force at ropes
FCS = closing force at ropes
Ratio = Ratio from pinion to rack
Radius to ropes = Rope pitch radius on sheave
n = efficiency from gages to trunnion

$$Ratio := \frac{280}{19}$$

$$Radius_{toropes} := \frac{15}{2}$$

$$n := .98$$

$$FOS_{ch1} := TOS_{ch1} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right) \qquad FOS_{ch2} := TOS_{ch2} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right)$$

$$FCS_{ch1} := TCS_{ch1} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right) \qquad FCS_{ch2} := TCS_{ch2} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right)$$

$$FOS_{ch3} := TOS_{ch3} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right) \qquad FOS_{ch4} := TOS_{ch4} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right)$$

$$FCS_{ch3} := TCS_{ch3} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right) \qquad FCS_{ch4} := TCS_{ch4} \cdot \left(\frac{Ratio \cdot n}{Radius_{toropes}}\right)$$

$$FO := FOS_{ch1} + FOS_{ch2} + FOS_{ch3} + FOS_{ch4}$$

$$FC := FCS_{ch1} + FCS_{ch2} + FCS_{ch3} + FCS_{ch4}$$

$$Imb_{total} := \frac{FO + FC}{2}$$

$$Friction := \frac{FO - FC}{2}$$

Equation for the fitting curve:

$$F(\text{Imbalance}_{set}, \text{height}) := \left[\text{Imbalance}_{set} + \left[-2\cdot\text{corner}\cdot(\text{Ropes}\cdot\text{Mass}_{unit})\cdot\text{height}\right] + \text{corner}\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\text{atan}\left(\frac{y_{aux}-\text{height}}{x_{aux}}\right) + \text{asin}\left[\frac{R_{aux}}{\left[\frac{x_{aux}}{\cos\left(\text{atan}\left(\frac{y_{aux}-\text{height}}{x_{aux}}\right)\right)}\right]}\right]\right]\right]\right]\right]$$

$i := 12 .. \text{length}(\text{height}) - 11$ i is a range variable for the number of rows in the above matrices. The points

$$SSE(\text{Imbalance}_{set}) := \sum_i \left(\text{Imb}_{total_i} - F(\text{Imbalance}_{set}, \text{height}_i)\right)^2$$

$\text{Imbalance}_{set} := 50000$, initial guess

Given

$SSE(\text{Imbalance}_{set}) = 0$

$\text{Imbalance}_{set} := \text{MinErr}(\text{Imbalance}_{set})$

$\text{Imbalance}_{set} = 41698.145$, Solution

$\text{height} := 0$

$$\text{Imbalance}_{ini} := \left[\text{Imbalance}_{set} + \left[-2\cdot(\text{corner}\cdot\text{Ropes}\cdot\text{Mass}_{unit})\cdot\text{height}\right] + \left[\text{corner}\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\text{atan}\left(\frac{y_{aux}-\text{height}}{x_{aux}}\right) + \text{asin}\left[\frac{R_{aux}}{\left[\frac{x_{aux}}{\cos\left(\text{atan}\left(\frac{y_{aux}-\text{height}}{x_{aux}}\right)\right)}\right]}\right]\right]\right]\right]\right]\right]$$

$\text{Imbalance}_{ini} = 7628$ $\text{mean}(\text{Friction}) = 4518$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

APPENDIX D

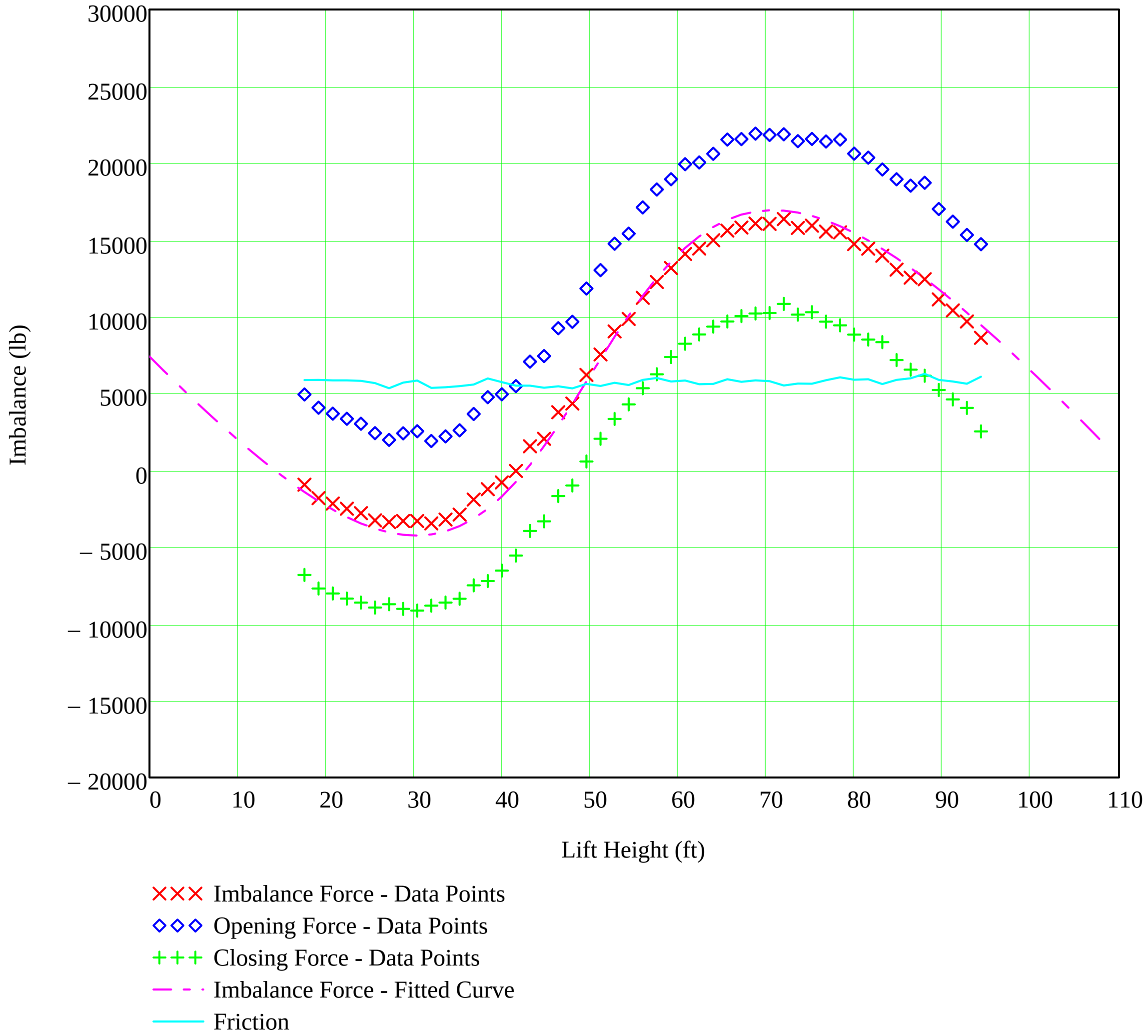
Graphical Results
Best Fit of Theoretical Imbalance Curve to Imbalance Data

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"

TOWER = "North Tower"

TEST_DATE = "May 10, 2017"   TEST_ID = "Run 1"



Average
100 Points

Analyze

$height_{min} = 17.6$

$height_{max} = 94.3$

$height_{ini} := 0$

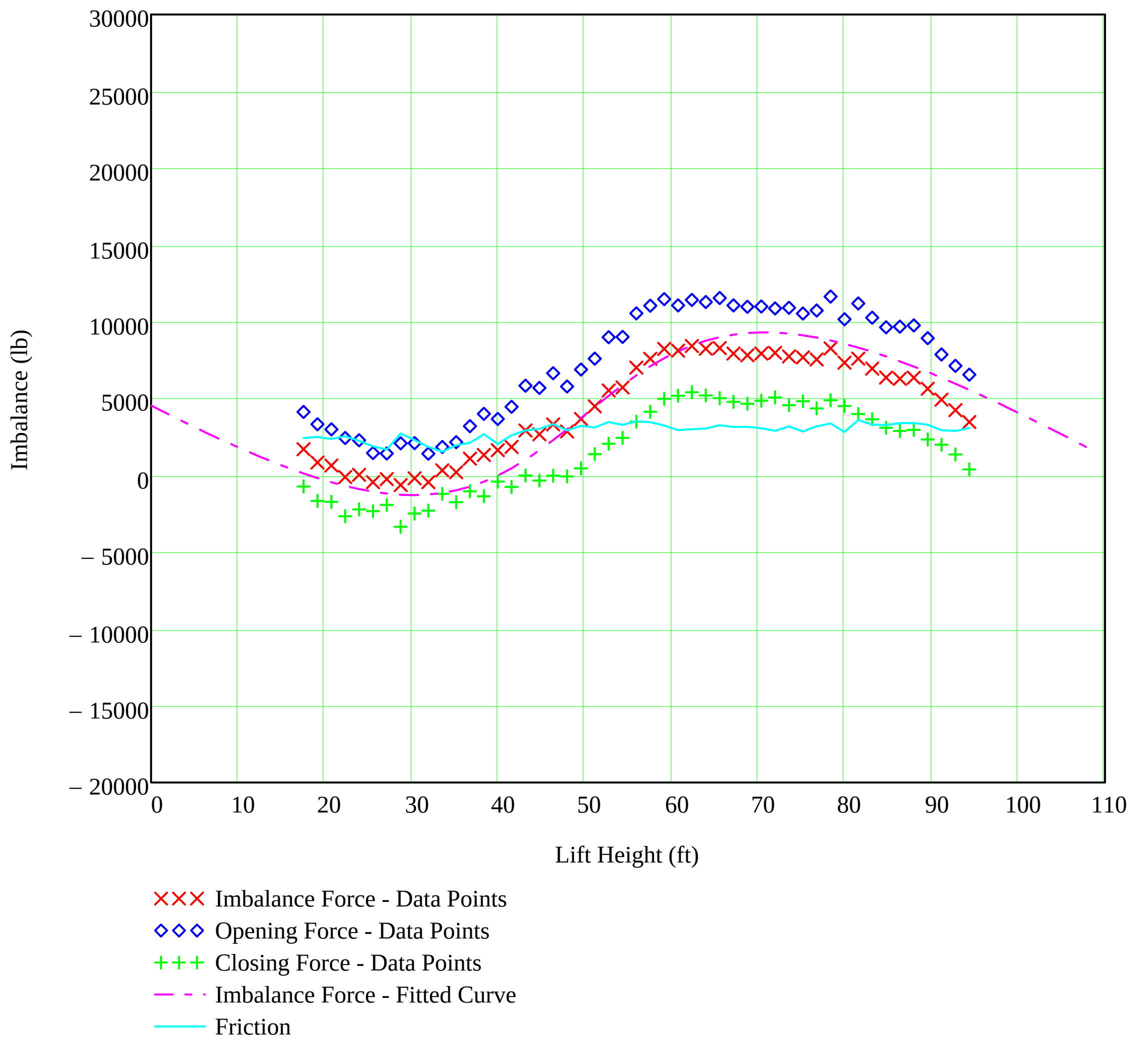
$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot(corners\cdot Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 7407$ $Friction_{ave} = 5731$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
Corner1 = "NW Corner"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 1"



Average
100 Points

Analyze

$height_{min} = 17.6$

$height_{max} = 94.3$

$height_{ini} := 0$

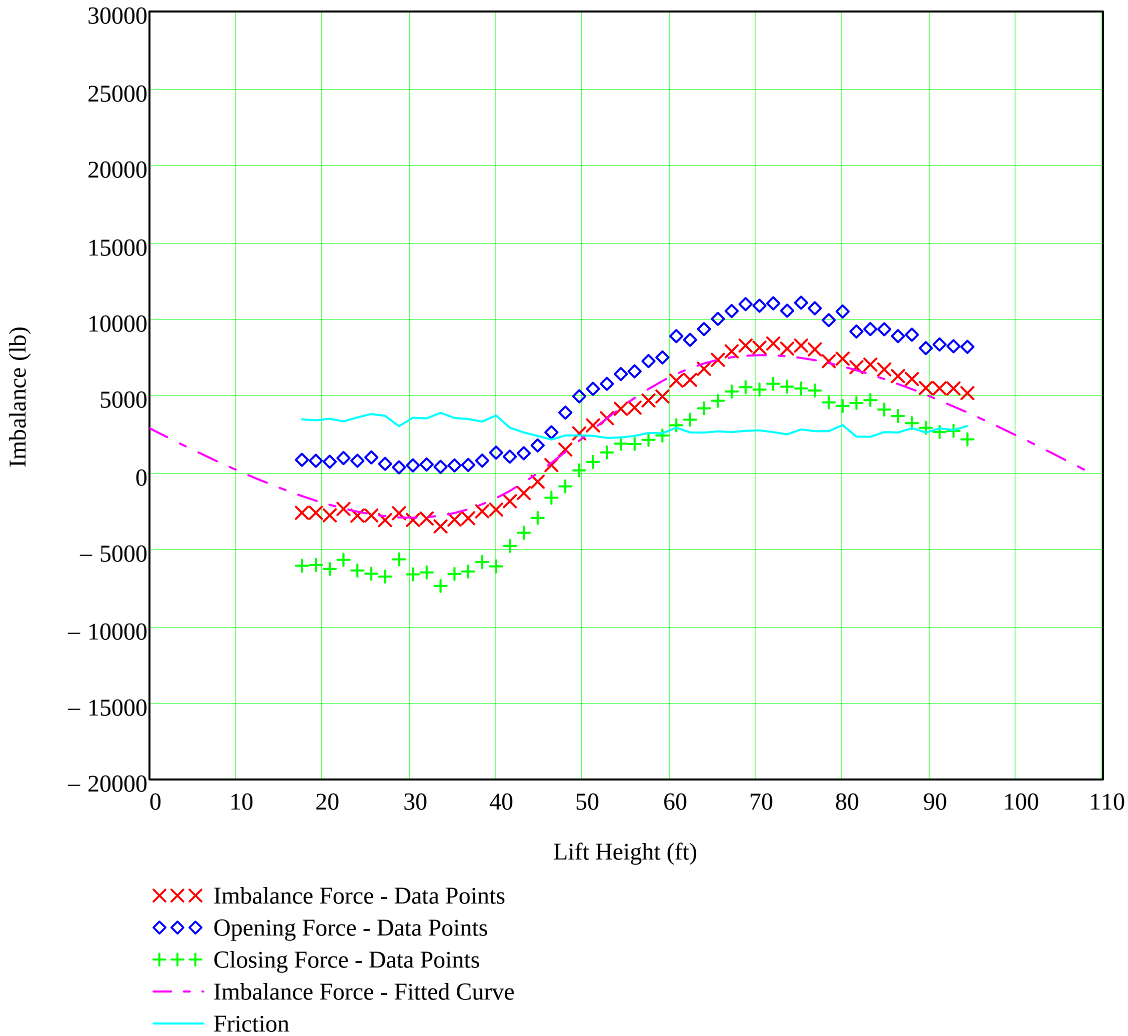
$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC1} = 4539$ $Friction_{aveC1} = 2857$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"
Corner2 = "NE Corner"
TEST_DATE = "May 10, 2017"   TEST_ID = "Run 1"



Average
100 Points

Analyze
$height_{min} = 17.6$
$height_{max} = 94.3$

$height_{ini} := 0$

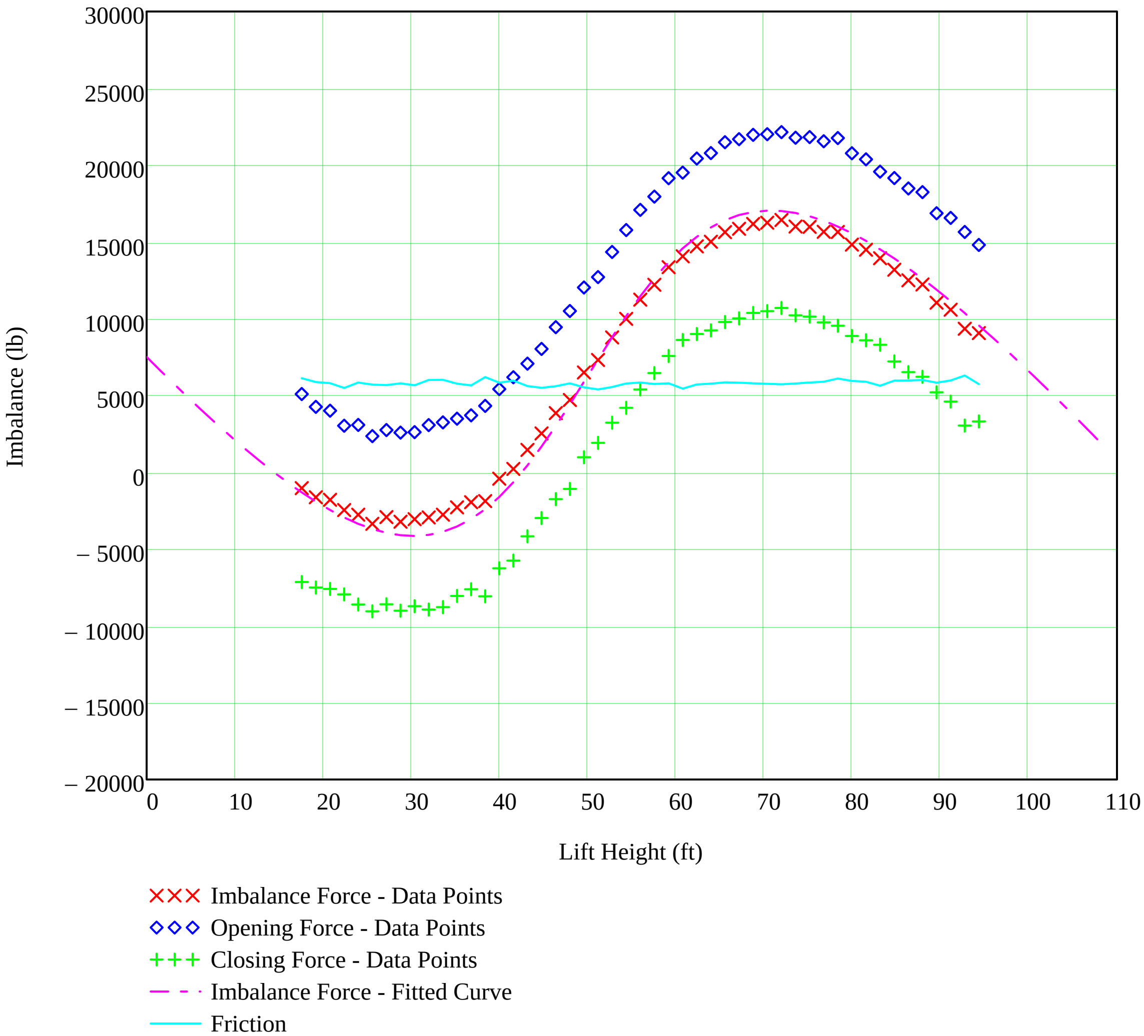
$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2\cdot(Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC2} = 2868$ $Friction_{aveC2} = 2873$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
TOWER = "North Tower"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 2"



Average
100 Points

Analyze

$height_{min} = 17.6$

$height_{max} = 94.3$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot\left(corners\cdot Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \right.$$
$$\left. + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 7512$ $Friction_{ave} = 5804$

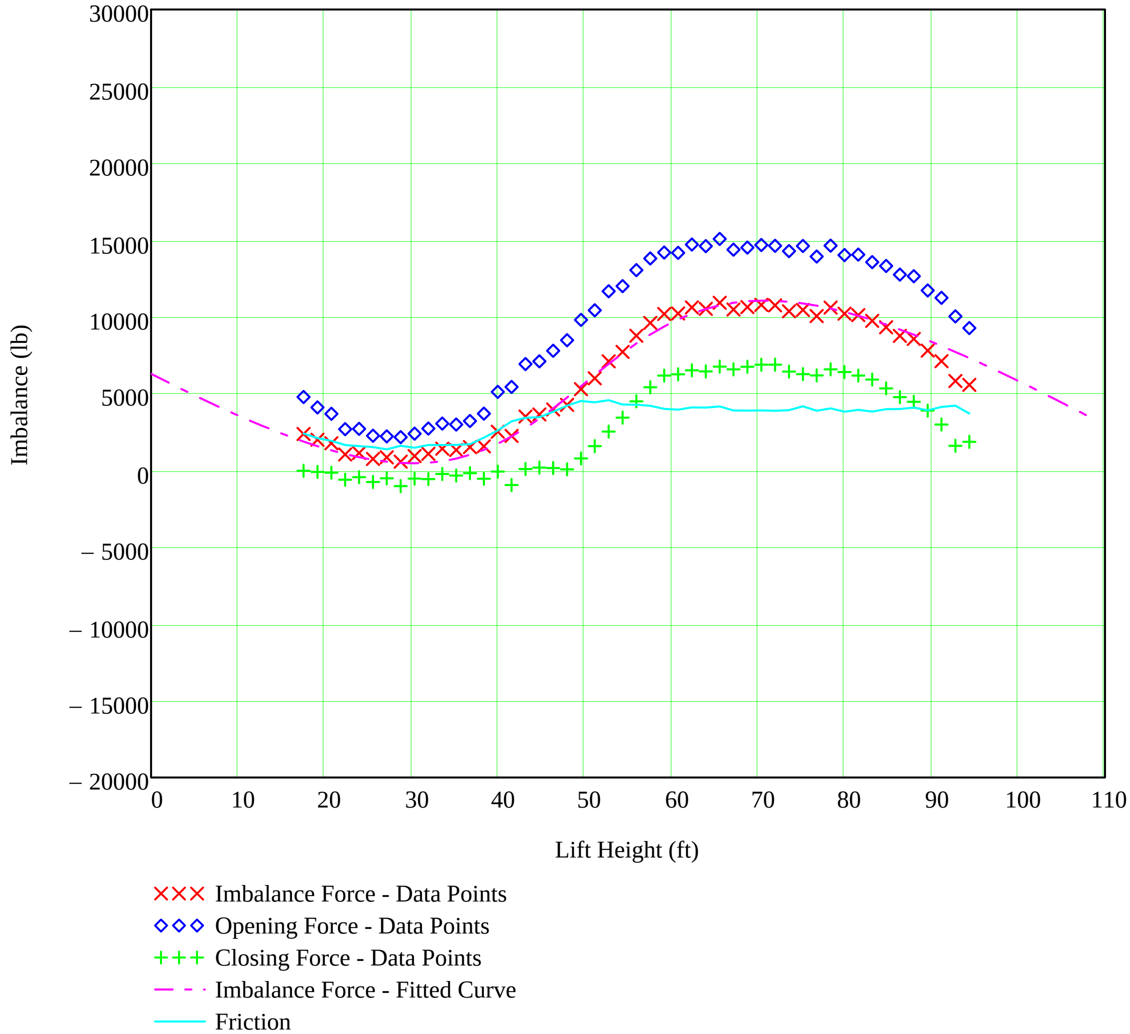
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner1 = "NW Corner"

TEST_DATE = "May 10, 2017"   TEST_ID = "Run 2"



Average
100 Points

Analyze

$height_{min} = 17.6$

$height_{max} = 94.3$

$height_{ini} := 0$

$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \\ + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\dfrac{x_{aux}}{\cos\left(\operatorname{atan}\left(\dfrac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

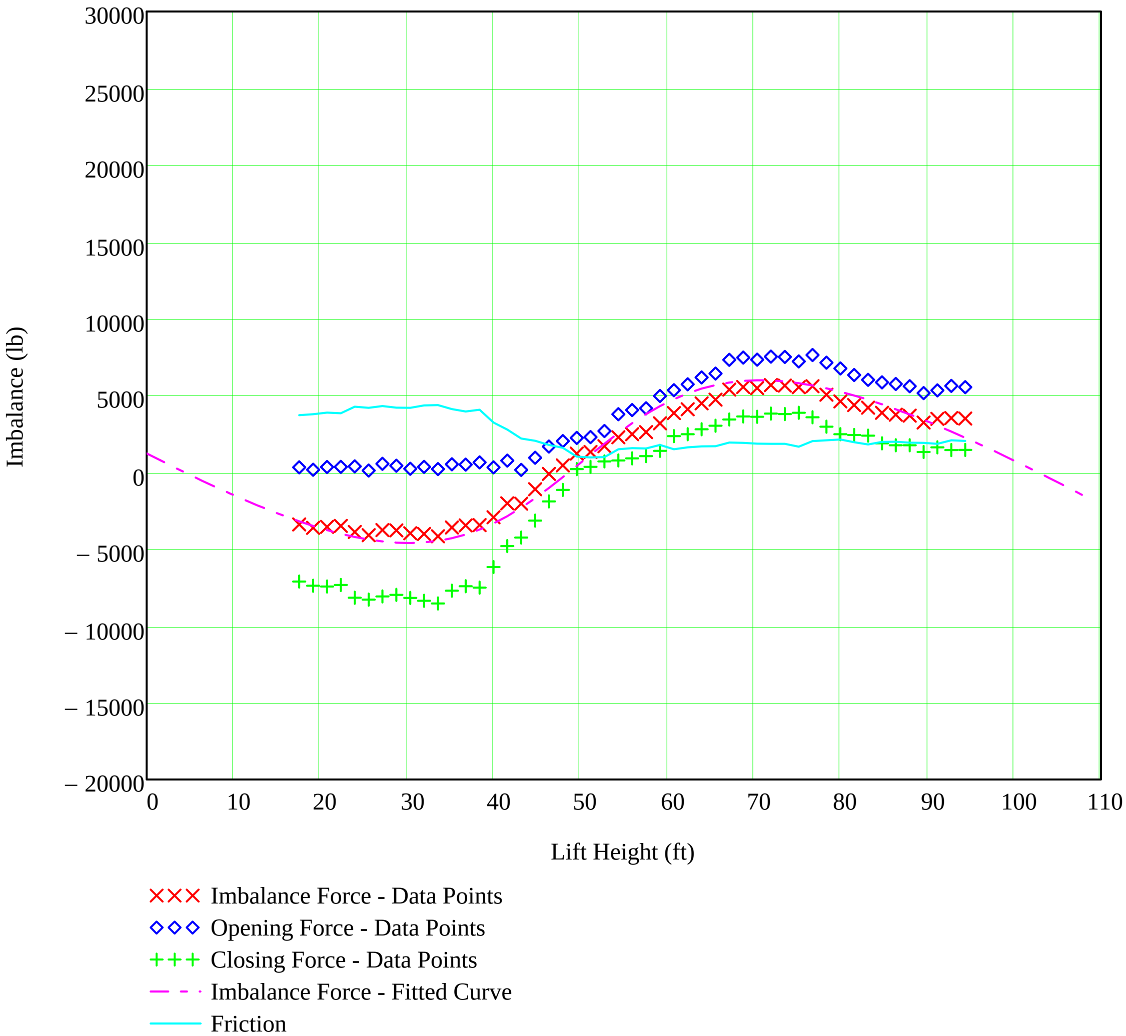
$Imbalance_{iniC1} = 6285$ $Friction_{aveC1} = 3319$

Positive imbalance indicates bridge is span heavy.

Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
Corner2 = "NE Corner"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 2"



Average
100 Points

Analyze

$height_{min} = 17.6$

$height_{max} = 94.3$

$height_{ini} := 0$

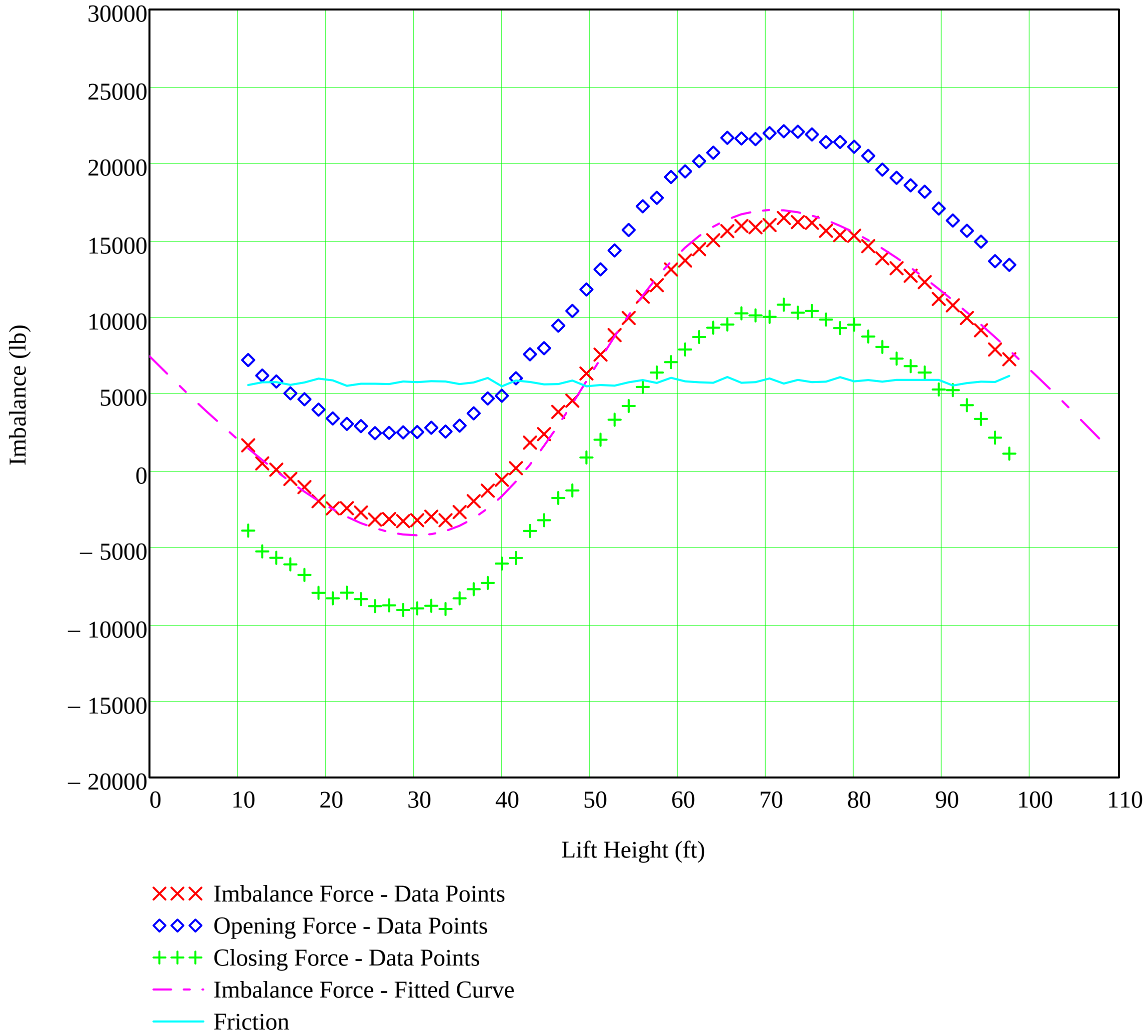
$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2\cdot(Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots \atop + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC2} = 1226$ $Friction_{aveC2} = 2485$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
TOWER = "North Tower"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 3"



Average
100 Points

Analyze
$height_{min} = 11.2$
$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot\left(corners\cdot Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \right.$$
$$\left. + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 7430$ $Friction_{ave} = 5756$

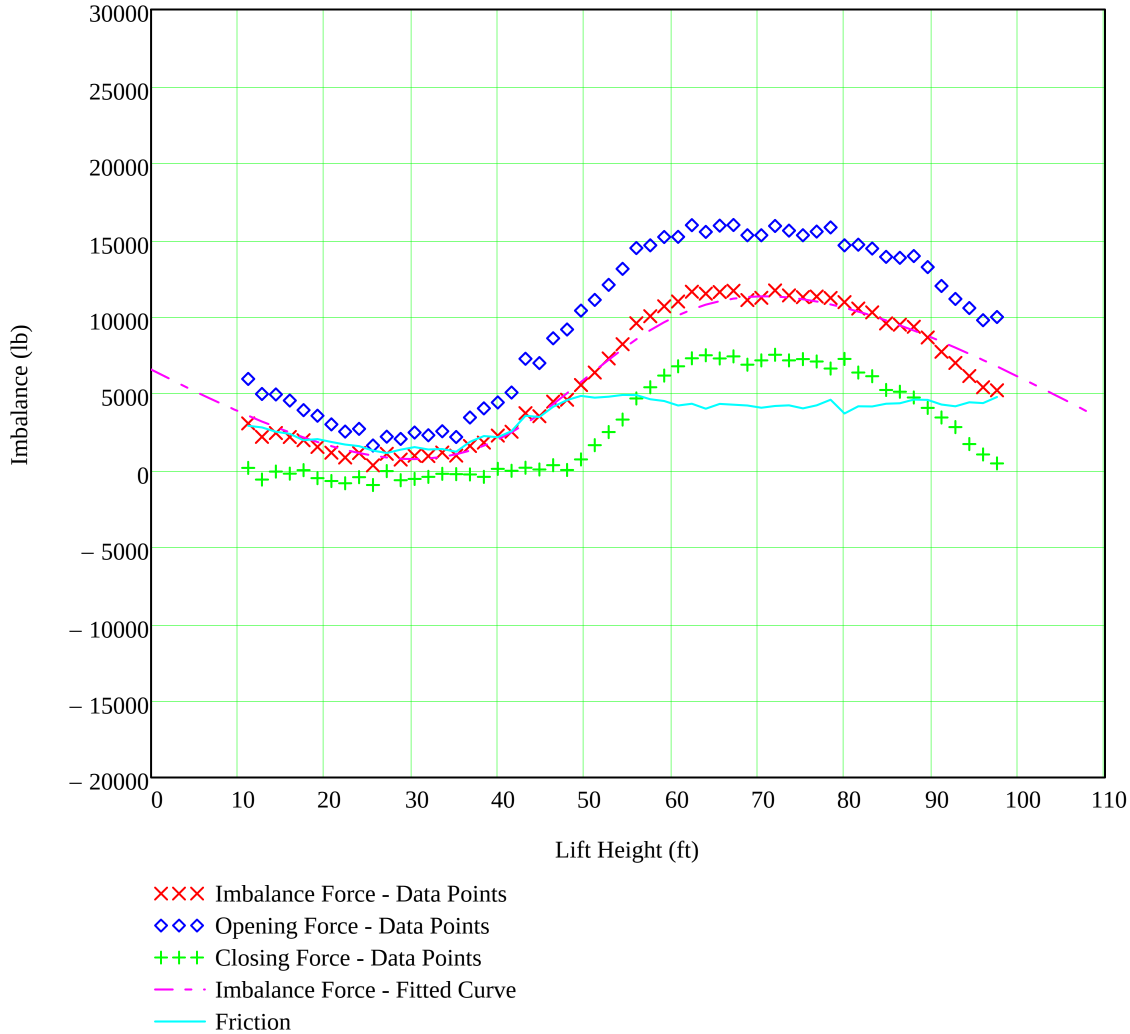
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner1 = "NW Corner"

TEST_DATE = "May 10, 2017"   TEST_ID = "Run 3"



Average
100 Points

Analyze

$height_{min} = 11.2$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \cdots + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\dfrac{x_{aux}}{\cos\left(\operatorname{atan}\left(\dfrac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC1} = 6559$ $Friction_{aveC1} = 3442$

Positive imbalance indicates bridge is span heavy.

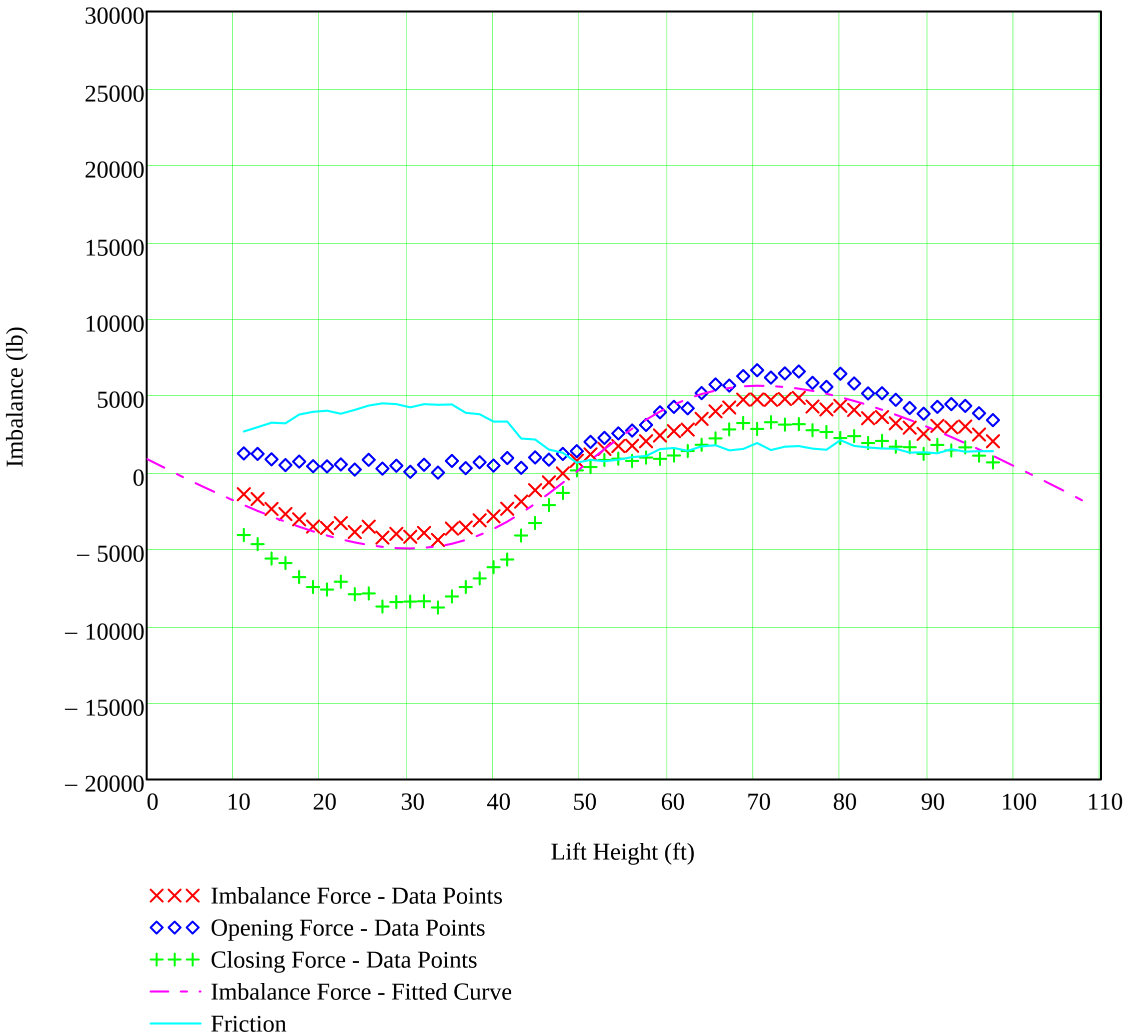
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner2 = "NE Corner"

TEST_DATE = "May 10, 2017" TEST_ID = "Run 3"



Average
100 Points

Analyze

$height_{min} = 11.2$

$height_{max} = 97.5$

$height_{ini} := 0$

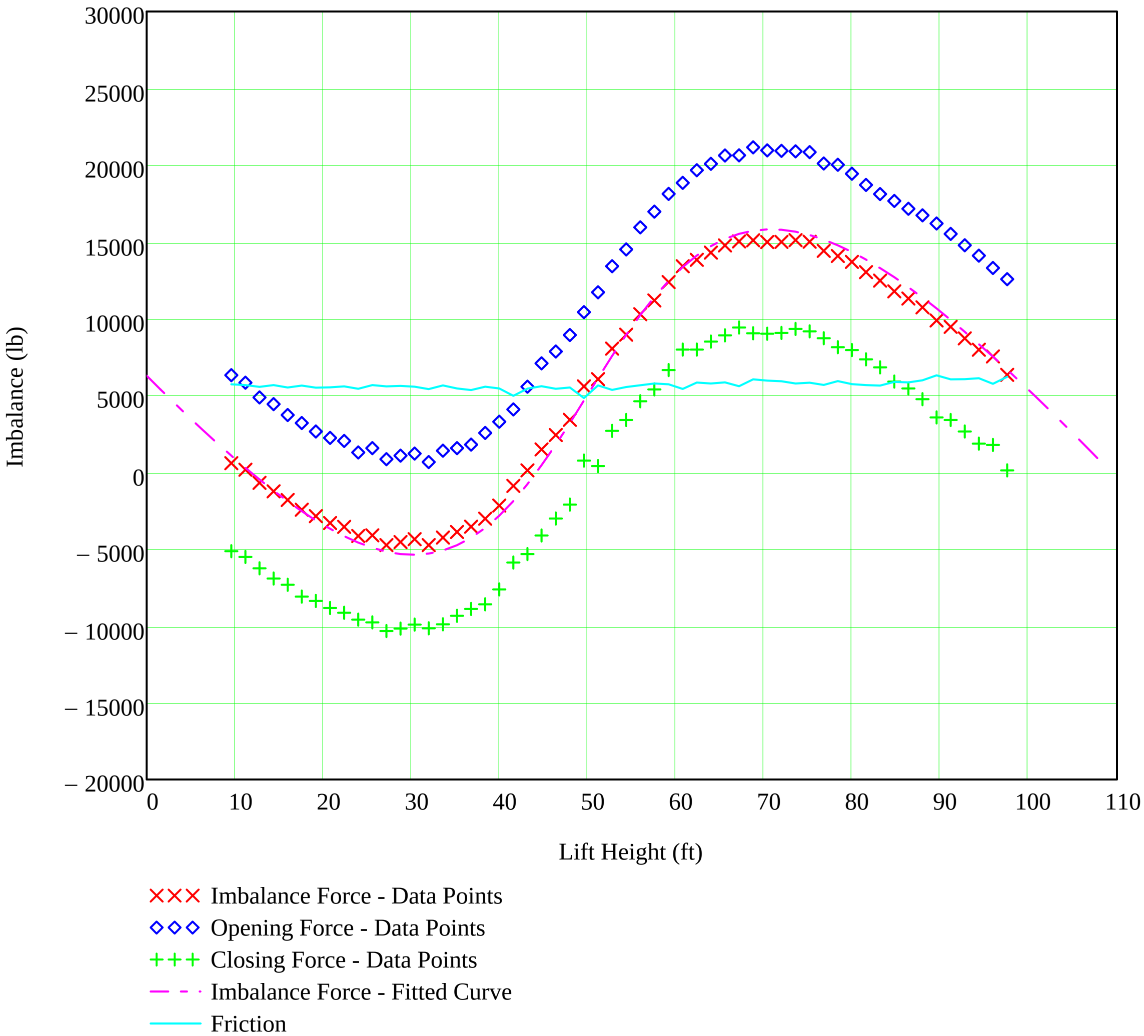
$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \atop + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]$$

$Imbalance_{iniC2} = 871$ $Friction_{aveC2} = 2314$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"
TOWER = "South Tower"
TEST_DATE = "May 10, 2017"   TEST_ID = "Run 1"



Average
100 Points

Analyze
$height_{min} = 9.6$
$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot\left(corners\cdot Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \\ + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 6291$ $Friction_{ave} = 5676$

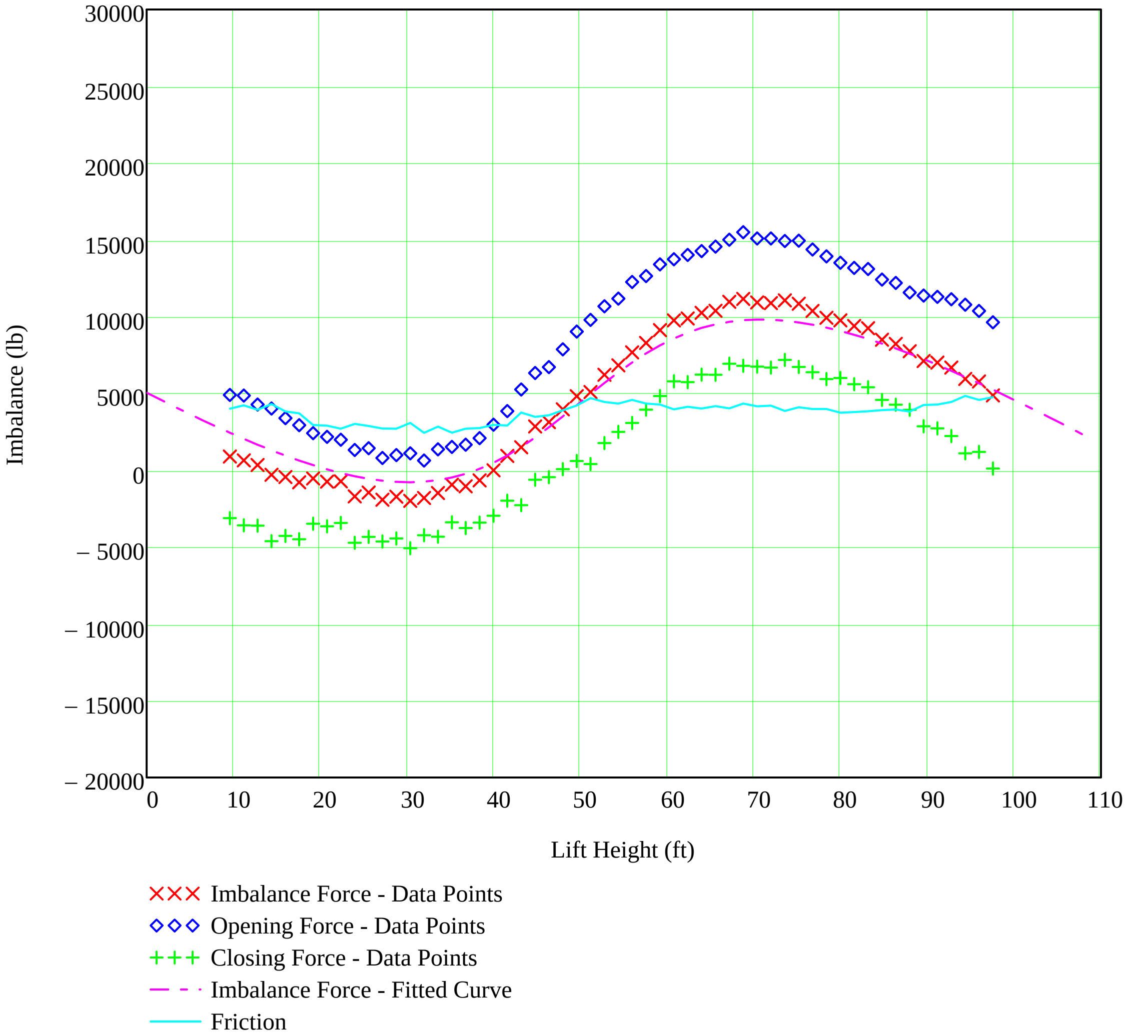
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner1 = "SW Corner"

TEST_DATE = "May 10, 2017" TEST_ID = "Run 1"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \\ + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\dfrac{x_{aux}}{\cos\left(\operatorname{atan}\left(\dfrac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC1} = 5050$ $Friction_{aveC1} = 3768$

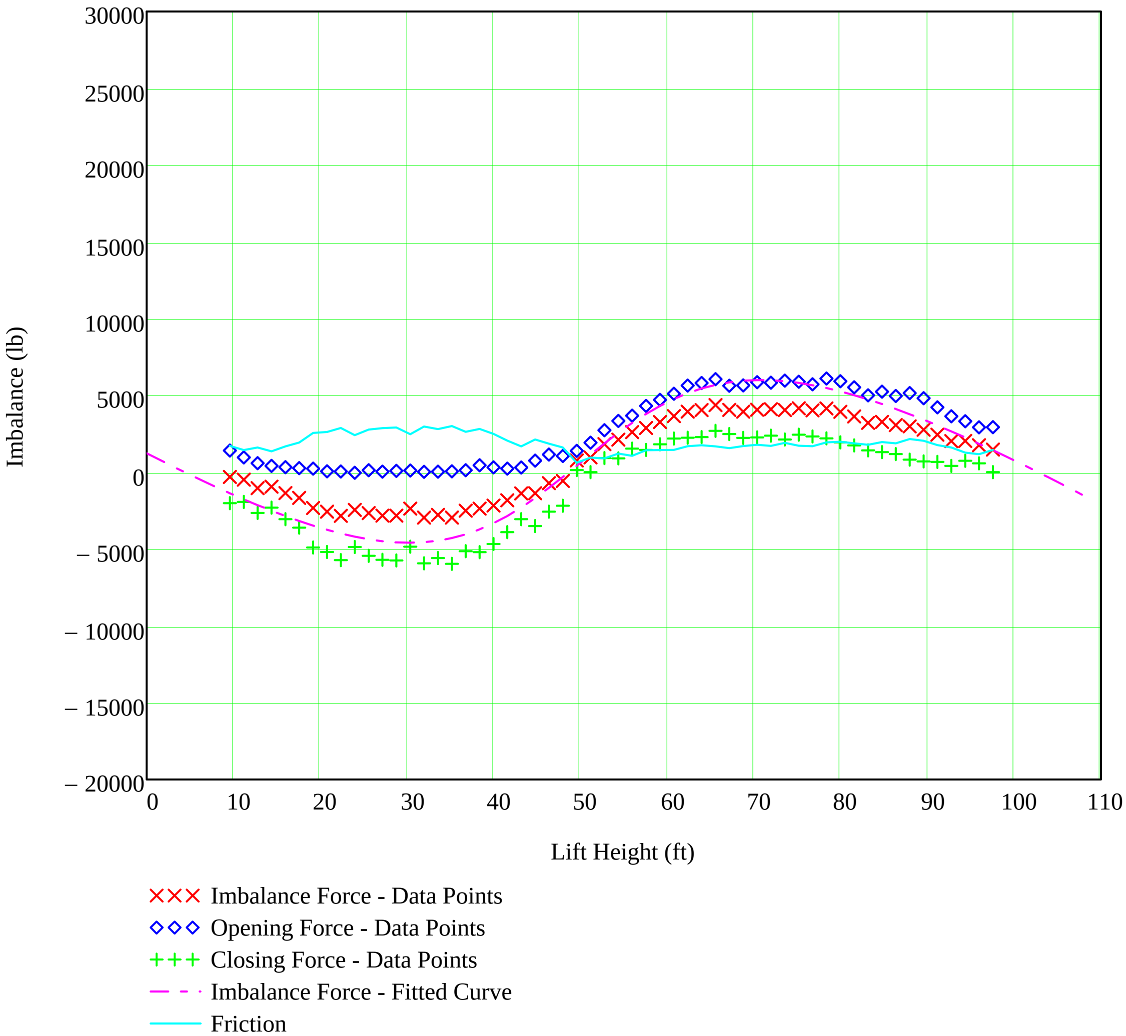
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner2 = "SE Corner"

TEST_DATE = "May 10, 2017" TEST_ID = "Run 1"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

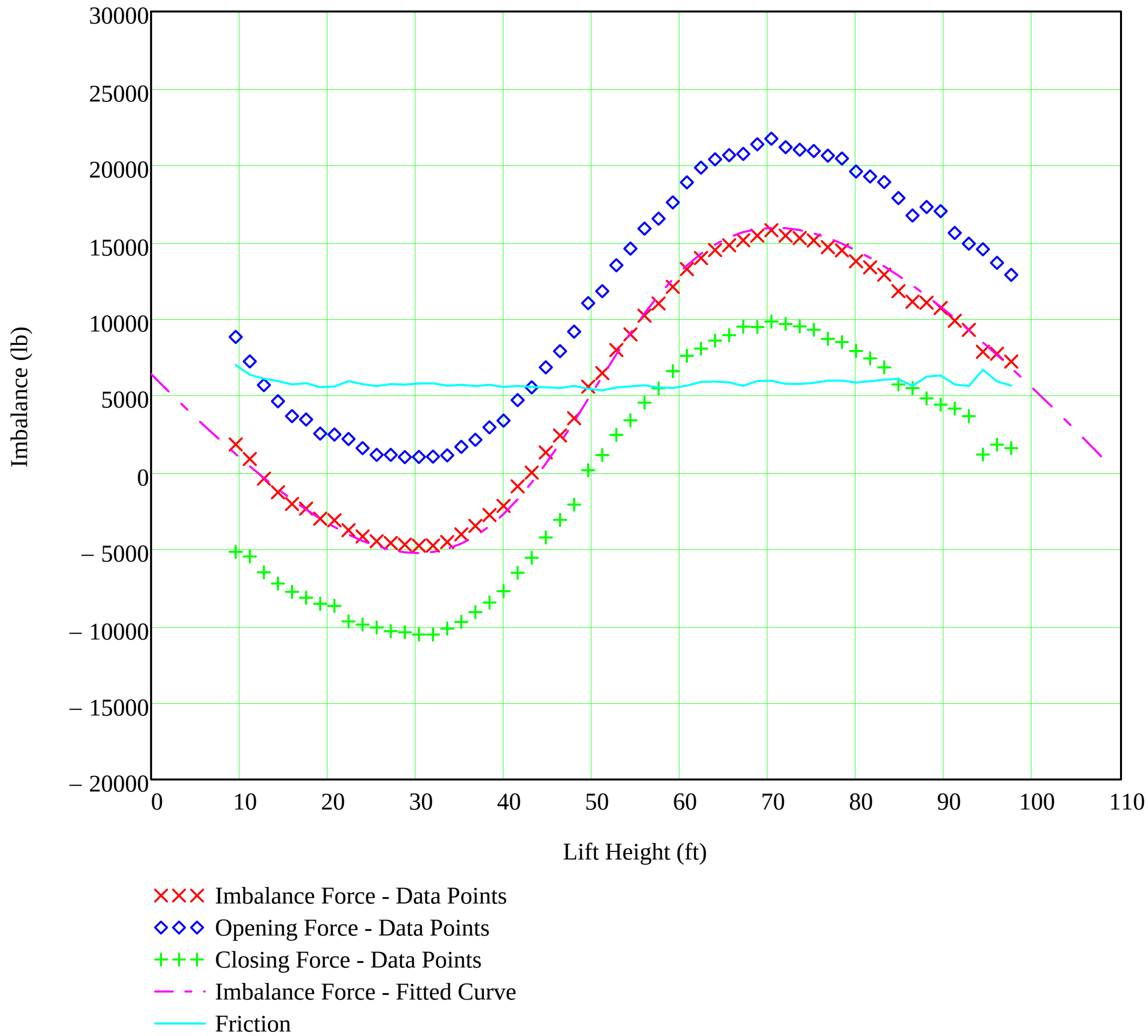
$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2 \cdot (Ropes \cdot Mass_{unit}) \cdot height_{ini} \right] \ldots \\ + corner \cdot \left[(-1) \cdot F_{cwt} \cdot \left[\sin \left[\operatorname{atan} \left(\frac{y_{aux} - height_{ini}}{x_{aux}} \right) + \operatorname{asin} \left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos \left(\operatorname{atan} \left(\frac{y_{aux} - height_{ini}}{x_{aux}} \right) \right)} \right)} \right] \right] \right] \right] \right]$$

$Imbalance_{iniC2} = 1241$ $Friction_{aveC2} = 1908$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
TOWER = "South Tower"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 2"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot(corners\cdot Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 6399$ $Friction_{ave} = 5796$

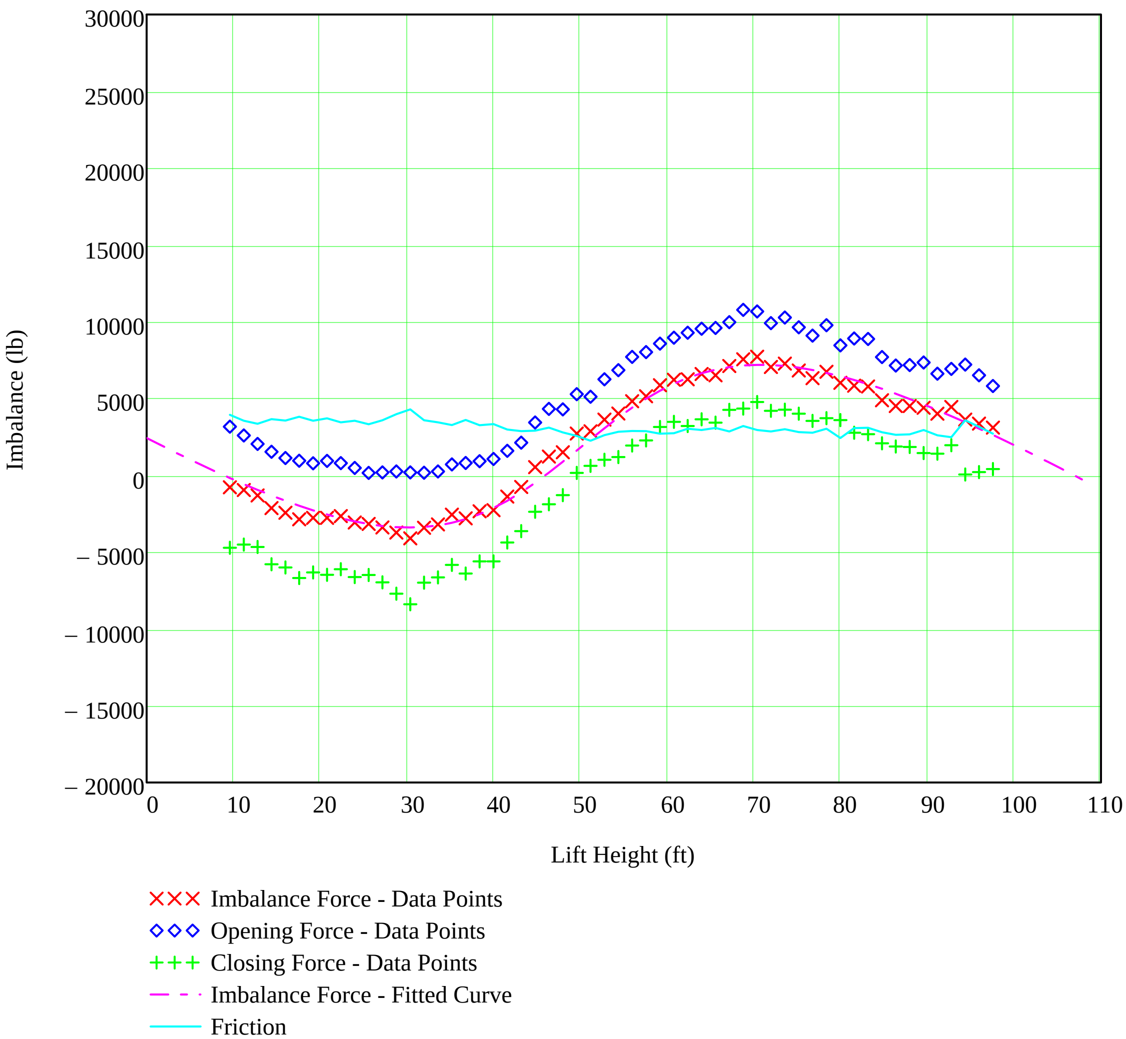
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner1 = "SW Corner"

TEST_DATE = "May 10, 2017"   TEST_ID = "Run 2"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2 \cdot \left(Ropes \cdot Mass_{unit}\right) \cdot height_{ini}\right] \ldots \\ + corner \cdot \left[(-1) \cdot F_{cwt} \cdot \left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC1} = 2428$ $Friction_{aveC1} = 3119$

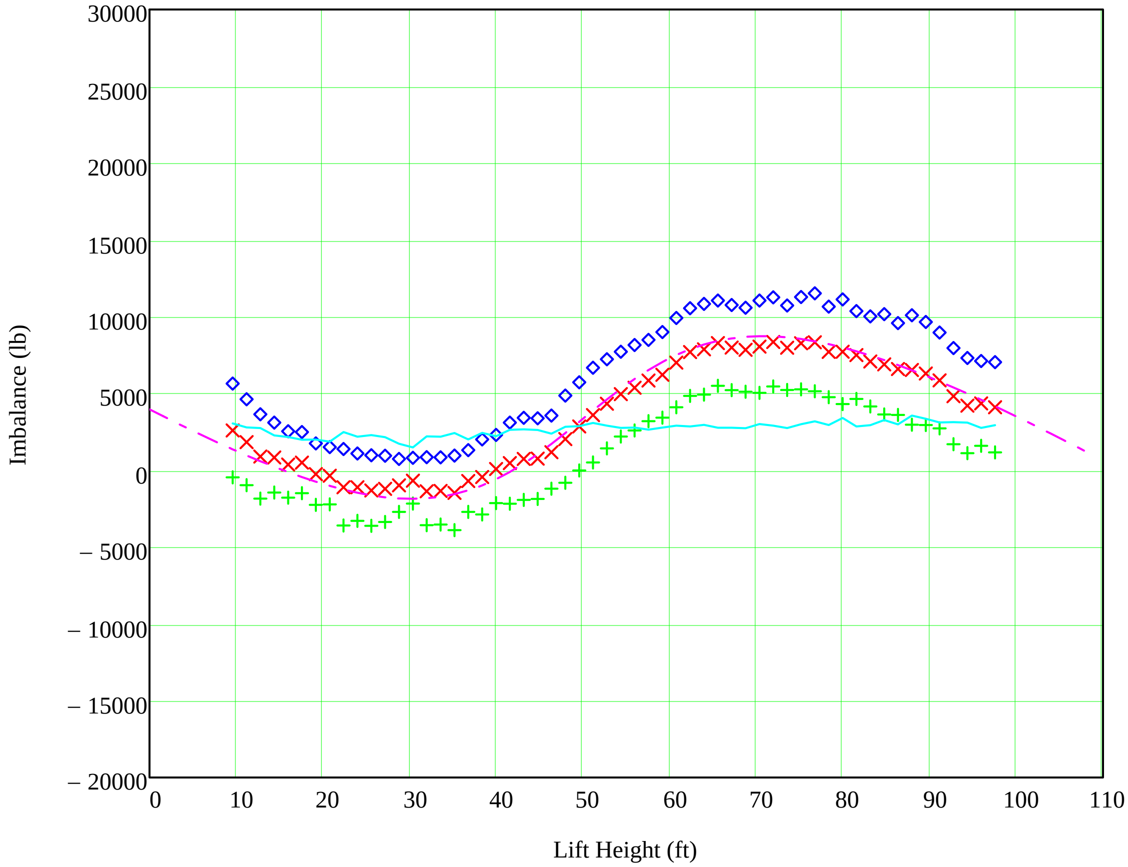
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"   BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner2 = "SE Corner"

TEST_DATE = "May 10, 2017"   TEST_ID = "Run 2"



××× Imbalance Force - Data Points
◇◇◇ Opening Force - Data Points
+++ Closing Force - Data Points
– – · Imbalance Force - Fitted Curve
—— Friction

Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

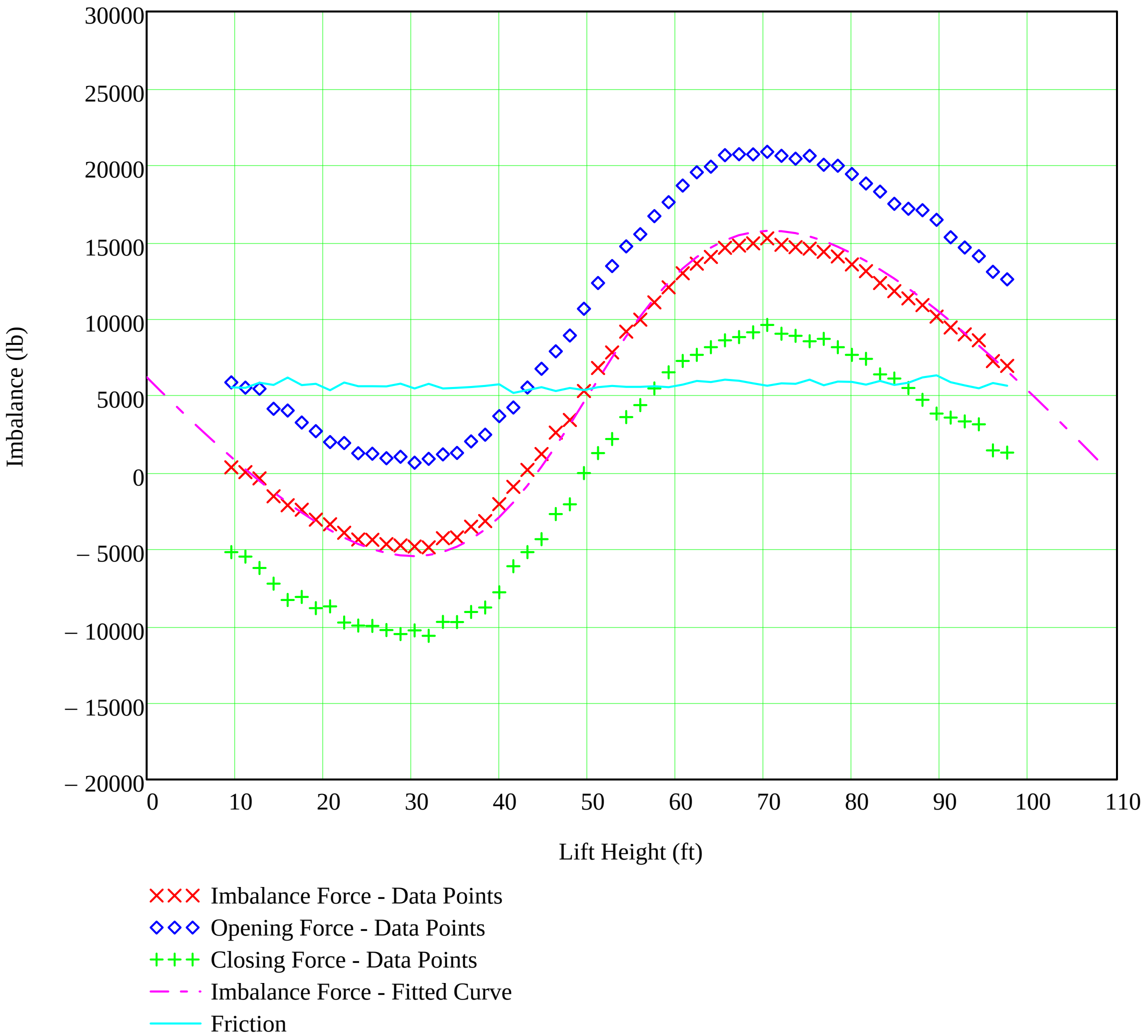
$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2\cdot(Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC2} = 3971$ $Friction_{aveC2} = 2677$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
TOWER = "South Tower"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 3"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot(corners\cdot Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

$Imbalance_{ini} = 6199$ $Friction_{ave} = 5692$

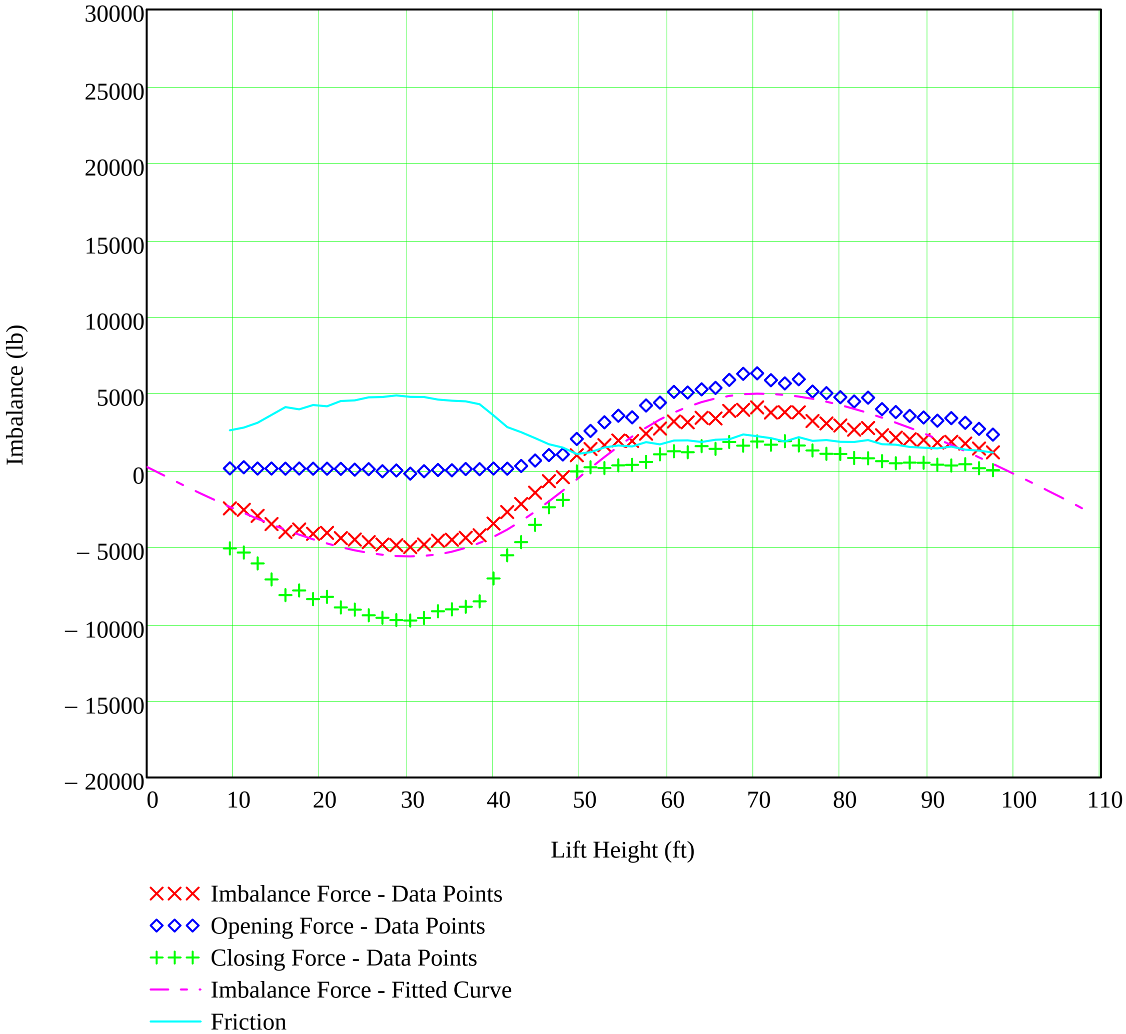
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"　　BRIDGE_TYPE = "Tower Drive Vertical Lift"

Corner1 = "SW Corner"

TEST_DATE = "May 10, 2017"　　TEST_ID = "Run 3"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

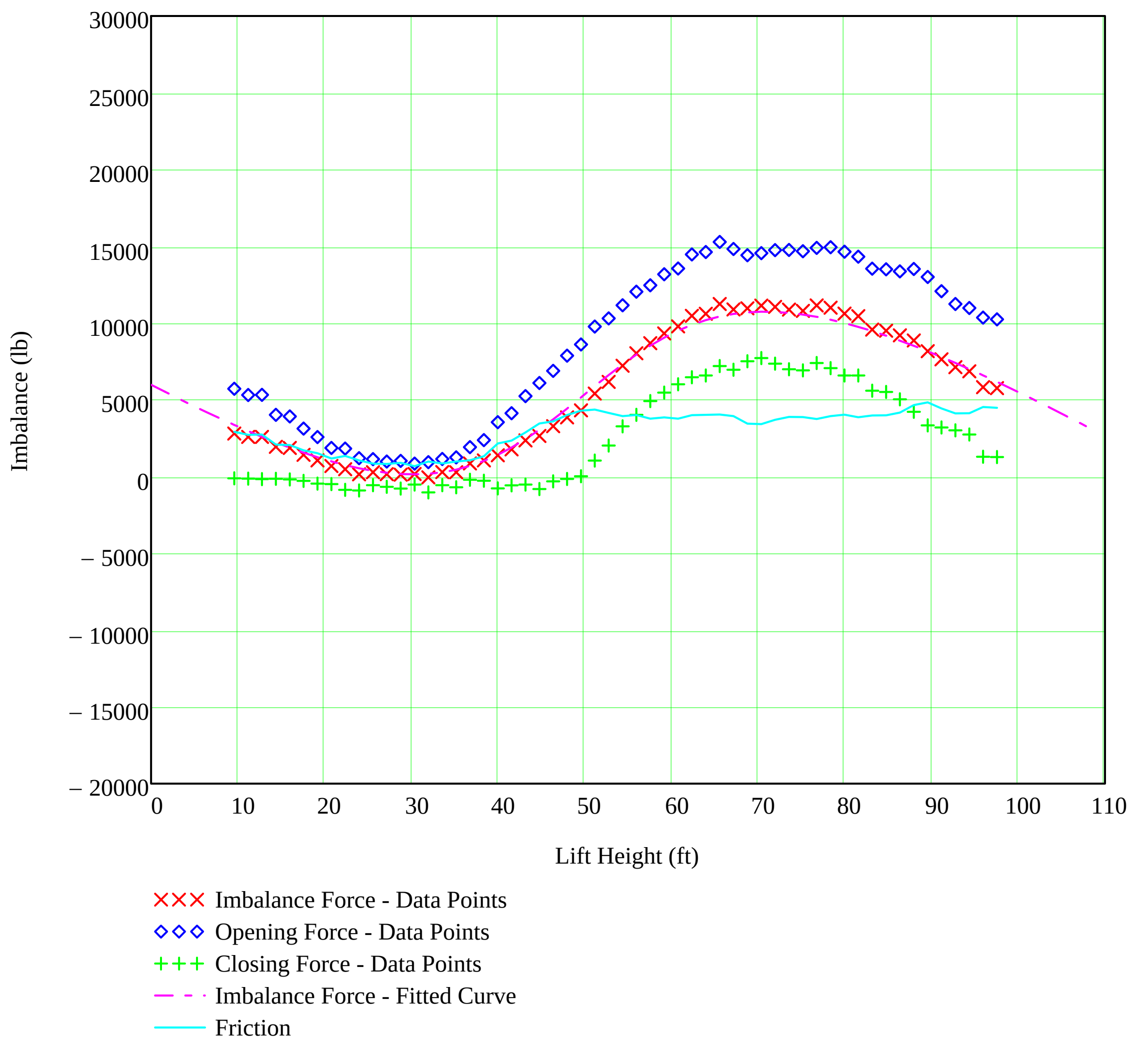
$$Imbalance_{iniC1} := \left[Imbalance_{setC1} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\dfrac{x_{aux}}{\cos\left(\operatorname{atan}\left(\dfrac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

$Imbalance_{iniC1} = 225$　　$Friction_{aveC1} = 2628$

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge" BRIDGE_TYPE = "Tower Drive Vertical Lift"
Corner2 = "SE Corner"
TEST_DATE = "May 10, 2017" TEST_ID = "Run 3"



Average
100 Points

Analyze
$height_{min} = 9.6$
$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{iniC2} := \left[Imbalance_{setC2} + \left[-2\cdot\left(Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \cdots \\ + corner\cdot\left[(-1)\cdot F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]$$

$Imbalance_{iniC2} = 5974$ $Friction_{aveC2} = 3064$

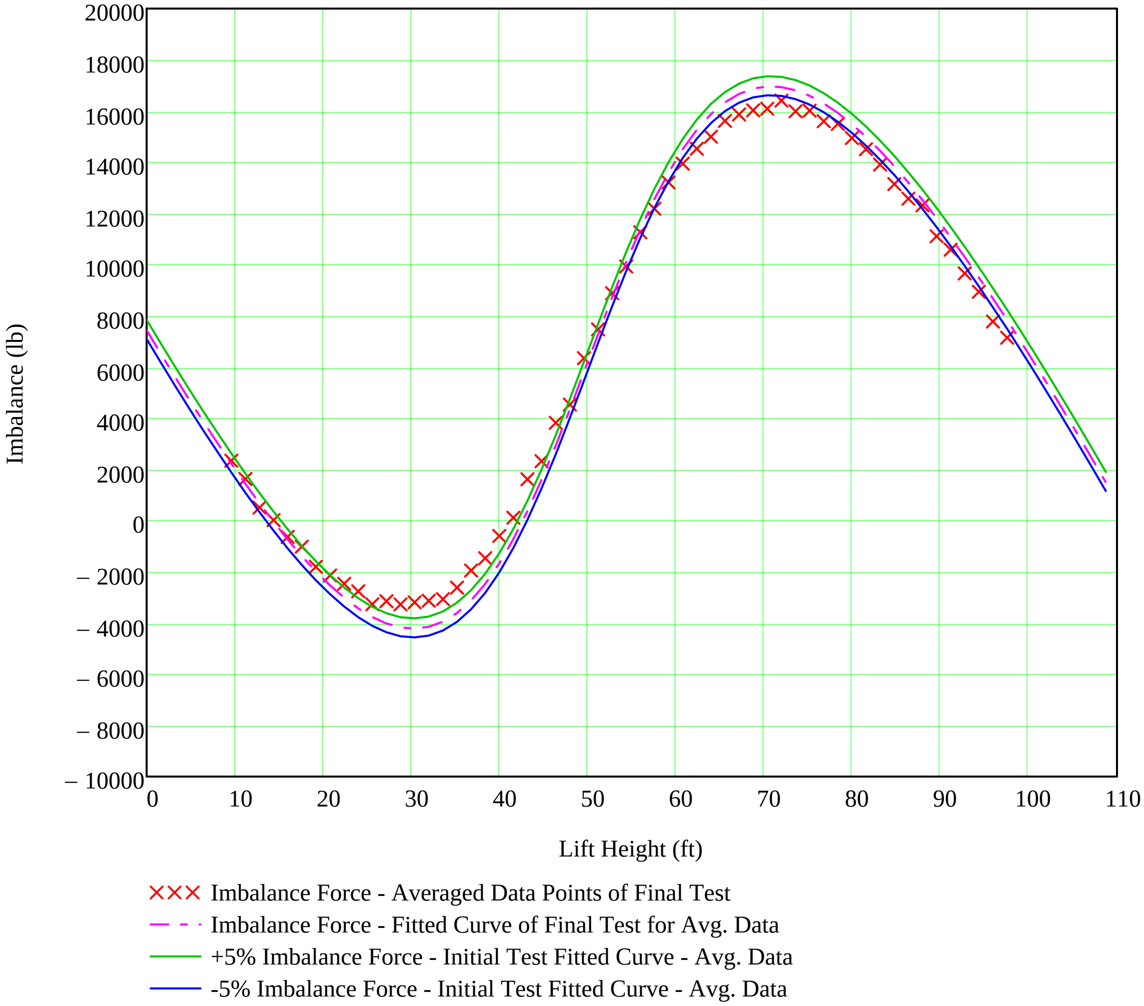
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"
TOWER = "North Tower"
TEST_DATE = "May 10, 2017"

BRIDGE_TYPE = "Tower Drive Vertical Lift"

TEST_ID = "Average Imbalance"



Average
100 Points

Analyze
$height_{min} = 9.6$
$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot\left(corners\cdot Ropes\cdot Mass_{unit}\right)\cdot height_{ini}\right] \ldots \\ + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]$$

May 10, 2017 North End Results
$Imbalance_{ini} = 7431$ lb. $Friction_{ave} = 5773$ lb.

April 22, 2015 North End Results
Initial Imblance = 7,452 lb. Friction = 5,755 lb.

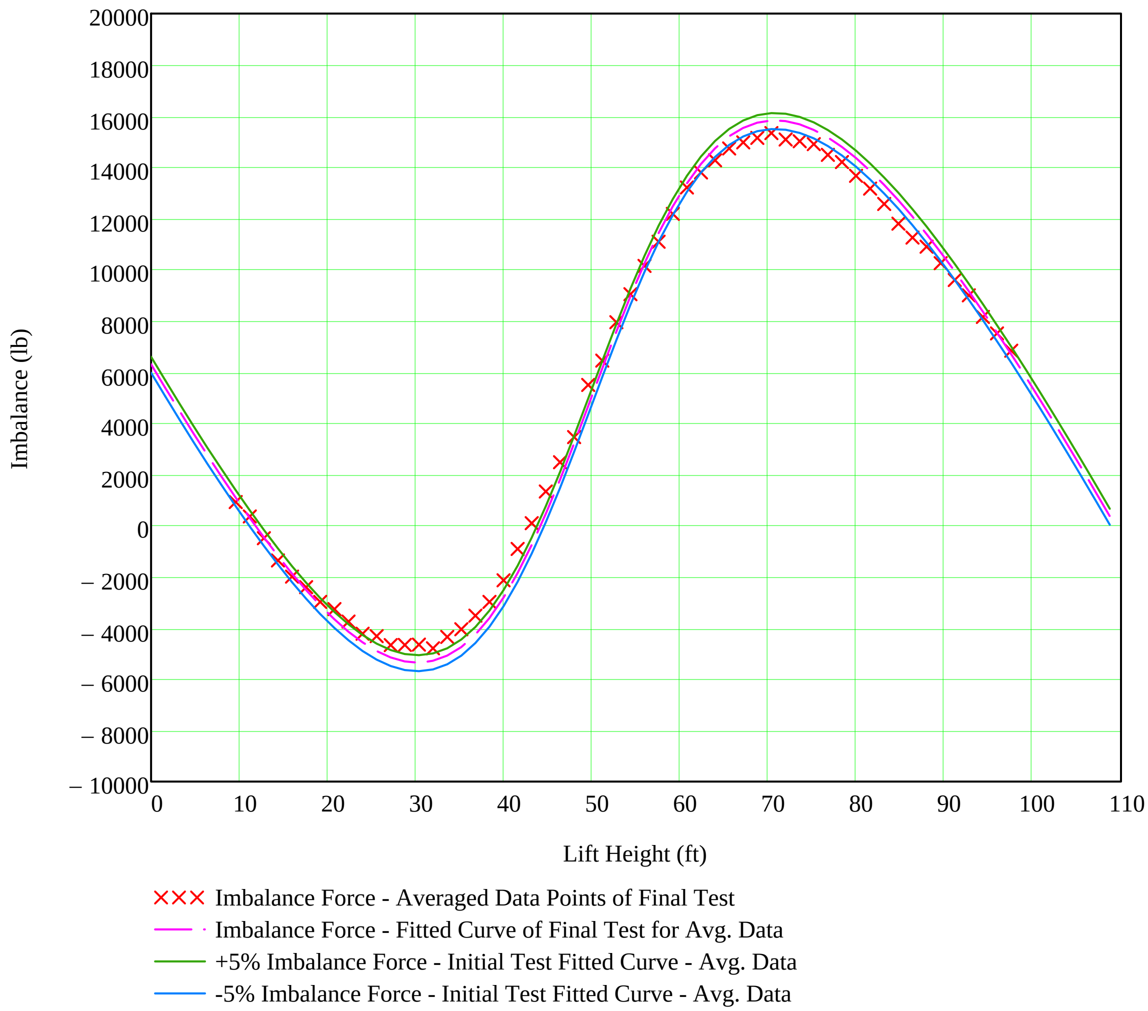
Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.

Span Balance Curves
Imbalance Versus Lift Height

BRIDGE = "Burlington Canal Lift Bridge"
TOWER = "South Tower"
TEST_DATE = "May 10, 2017"

BRIDGE_TYPE = "Tower Drive Vertical Lift"

TEST_ID = "Average Imbalance"



Average
100 Points

Analyze

$height_{min} = 9.6$

$height_{max} = 97.5$

$height_{ini} := 0$

$$Imbalance_{ini} := \left[Imbalance_{set} + \left[-2\cdot(corners\cdot Ropes\cdot Mass_{unit})\cdot height_{ini}\right] \ldots + \left[corners\cdot\left[-F_{cwt}\cdot\left[\sin\left[\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right) + \operatorname{asin}\left[\frac{R_{aux}}{\left(\frac{x_{aux}}{\cos\left(\operatorname{atan}\left(\frac{y_{aux} - height_{ini}}{x_{aux}}\right)\right)}\right)}\right]\right]\right]\right]\right]\right]$$

May 10, 2017 South End Results
$Imbalance_{ini} = 6296$ lb. $Friction_{ave} = 5721$ lb.

April 22, 2015 South End Results
Initial Imblance = 6,274 lb. Friction = 5,215 lb.

Positive imbalance indicates bridge is span heavy.
Negative imbalance indicates bridge is counterweight heavy.